VOLUME XI SECOND SERIES NUMBER 3

# THE PHYSICAL REVIEW

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PUBLISHED BY

THE AMERICAN PHYSICAL SOCIETY

LANCASTER, PA., AND ITHACA, N. Y.

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*Second Series.* *March, 1918* *Vol. XI., No. 3*

THE

# PHYSICAL REVIEW.

## THE MAGNETIZATION OF IRON IN THE ABSENCE OF HYSTERESIS.

By Winthrop R. Wright.

ANY investigation of the magnetic properties of ferro-magnetic substances is complicated by the presence of hysteresis. Even the curve, usually known as the magnetization curve, is, as Steinmetz[1] points out, but one side of an unsymmetrical hysteresis loop, and differs from any other loop only in passing through the origin. The advantages to be gained in suppressing hysteresis are evident. Without hysteresis, the magnetization becomes a single-valued function of the magnetizing field and it is feasible to attempt an equation connecting them. Again, the true effect of the temperature upon magnetization may be investigated, for the effect of temperature upon hysteresis is so marked that its true effect upon magnetization may be entirely masked, especially in the case of weak fields.

In general, two methods have been proposed for suppressing hysteresis, mechanical shocks or vibrations and an alternating magnetic field, either transverse or longitudinal. Ewing[2] employed mechanical vibrations while Finzi,[3] Ashworth,[4] and Steinhaus and Gumlich[5] used alternating fields superposed upon the magnetizing field. Ashworth alone investigated the change of magnetization with the temperature, but his results, while free from hysteresis, were distorted by the alternating field which was present in the specimen. Steinhaus and Gumlich avoided this distortion by reducing the alternating field to zero before observing the magnetization produced in the specimen due to the applied magnetizing field.

[1] Steinmetz, Theory and Calculation of Electric Circuits, p. 50.
[2] Ewing, Phil. Trans., p. 564, 1885.
[3] Finzi, Electrician, 26, 672, 1891.
[4] Ashworth, Phil. Mag., 27, 357, 1914.
[5] Steinhaus and Gumlich, Ber. d. Deut. Phys. Ges., 17, 369, 1915.

The present work is, in a sense, a repetition of Ashworth's work on iron, using the method of Steinhaus and Gumlich. Five specimens were prepared from samples furnished through the kindness of Professor E. D. Campbell. These included three hypo-eutectoid steels, a very pure basic open-hearth steel, and an ingot iron. Their composition was furnished with them and appears in Table I. These five form a series of steels with decreasing carbon content whose last member approximates pure iron. The specimens were made in the form of ellipsoids of revolution, 20 cm. long and 0.47 cm. in diameter.

TABLE I.

*Composition of Steels in Per Cent.*

| Steel. | C. | Mn. | P. | Si. | Cu. | S. |
|---|---|---|---|---|---|---|
| H57 | 0.57 | 0.11 | 0.010 | 0.17 | | 0.020 |
| H41 | 0.41 | 0.08 | 0.012 | 0.19 | | 0.016 |
| H35 | 0.35 | 0.08 | 0.009 | 0.18 | | 0.024 |
| 04 | 0.04 | 0.10 | 0.007 | | | 0.029 |
| INI | 0.015 | 0.016 | 0.005 | | 0.045 | 0.03 |

## APPARATUS.

A magnetometer was used for observing the magnetization of the specimen. Two identically wound solenoids were mounted east and west with the principal needle of the magnetometer on their common axis and between them. These solenoids were made of brass tubes, 4 cm. in diameter, with a layer, 60 cm. long, of No. 20 enamelled copper wire wound on them. They made available magnetizing fields up to 100 gauss in strength. The one solenoid, which was used for magnetizing the specimen, had a second layer wound on it, by which the required alternating field could be produced. Each solenoid was mounted in a copper tank which was water cooled. The second solenoid was used to balance the first and could be shifted longitudinally.

The magnetometer was of the astatic type devised by Kohlrausch and Holborn.[1] The moving system consisted of two sets of two needles each, 2.0 cm. long and 0.09 cm. in diameter, mounted at the ends of a glass rod, 70 cm. long and 0.1 cm. in diameter, and was suspended by a quartz fiber, 40 cm. long and 30 microns in diameter. Though the upper needles were slightly stronger, the instrument possessed a steady zero point and was sufficiently sensitive, a field of 0.00005 gauss causing a scale deflection of 2 mm. with a scale distance of 1.5 m. As the magnetometer was to be used in a null method, these were the only require-

[1] Kohlrausch and Holborn, Ann. d. Phys., 10, 287, 1903.

ments to be met and it was not necessary to ascertain to what extent the needle systems differed.

The magnetization produced in the specimen was measured by means of a coil, mounted on the side of the magnetometer opposite to the specimen and at an equal distance. By passing a suitable current through this coil, the deflection produced by the magnetization of the specimen was balanced and the needles of the magnetometer were brought back to their zero position, which was indicated by the familiar lamp, slit, mirror, and scale device. The magnetization of the specimen could then be calculated in terms of the current and the constants of the coil and the ellipsoid. The coil was made by winding No. 20 enamelled copper wire upon a core of Keene cement, a disc 17.3 cm. in diameter and 3.1 cm. thick. A slot was cut on the rim of the disc and a single layer of wire wound on it. This layer was covered with more cement and a new surface was cut after the cement had hardened. On this new surface, a second layer was wound and the process was repeated until five layers had been put on, separated from each other by from two to four millimeters of cement. This method of assembling the coil permitted the accurate measurement of the dimensions of each layer and the field of the coil could be calculated from the formula for a single layer.

The data taken for a given magnetization curve involved the determination of the corresponding values of two currents, that through the magnetizing solenoid and that through the coil. This method for determining the magnetization seems to have much in its favor. It is independent of changes in the sensitivity of the indicating instrument and in the strength of the earth's field, even if the magnet systems are not exactly equal and are not accurately placed in the magnetic meridian. It replaces readings of a telescope and scale with those of a second ammeter, one having to be read for the magnetizing current, and thus affords two observations of the same type. It would seem that there is no difference in the rapidity with which observations can be taken since this depends so largely on the period and damping of the magnetometer in any method.

A 60-cycle alternating current, regulated by means of a water rheostat, was used in the outer winding of the magnetizing solenoid to produce the requisite alternating field. This rheostat had two electrodes of copper whose area was about 150 cm.$^2$ and, by lifting the movable electrode, the current could be reduced from about ten amperes to a few hundredths of an ampere before the final break occurred. It may be questioned whether such a device for reducing the current is legitimate, since it does involve a break in the current, though not until the latter

is small. By this means, however, the same magnetization was produced in the specimen for a given field, though the initial state was varied as widely as possible, and this should be a conclusive test for the absence of hysteresis. An objection to the rheostat may also be based upon its rectifying action due to inequality in the areas of the electrodes as the movable one is removed from the water. It was found, however, that the magnetization did not depend upon the direction of the rectified current through the solenoid. Evidently, when the alternating current became small enough to be neglected, the rectified portion was also neglible.

The specimen was heated by means of an electrical heater which fitted snugly within the brass tube of the magnetizing solenoid. The heating wires were of 25 per cent. nickel-steel and ran longitudinally, being held in place by alundum cement at equal spaces around the heating chamber. The latter was 60 cm. long and 1 cm. in diameter. A longitudinal winding produced no magnetic field within the heating space and secured a more uniform temperature throughout that part in which the specimen lay. The necessary thermal insulation was secured by two concentric quartz tubes, separated by asbestos, which slipped over the hollow alundum cylinder in which the wires were set. The heater was slightly magnetic below 500° C. but separate observations were taken to correct for this.

The ends of the magnetizing solenoid were provided with brass cover plates, made oil tight with asbestos gaskets. Through one plate passed a brass plug in which were mounted the tubes for a platinum resistance thermometer. The thermometer wire with its leads was stretched in a quartz tube, 1 mm. in bore, which was then bent double. The wire was long enough to traverse the length of the specimen twice, the latter being supported by the same tube which contained the wire. The compensating leads were mounted in a shorter piece of the same tubing. With such a thermometer the average temperature throughout the specimen was indicated.

## EXPERIMENTAL RESULTS.

The usual procedure with a specimen began with heating it for about an hour and a half in the neighborhood of the Curie point in order to anneal the specimen and to secure thermal equilibrium in the heater, solenoid, and oil bath. The temperature was then reduced slowly, step by step, and magnetization curves were taken at suitable intervals. The greatest change in magnetization occurred within the first one hundred degrees below the Curie point and from four to five hours were

allowed for this interval. About twice this time was taken for the specimen to cool completely to room temperature. Measurements were taken during cooling that they might be more free from irregularities due to previous thermal and mechanical treatment of the steels.

The magnetization curves obtained for specimen H35, the softest of the hypo-eutectoid steels, appear in Fig. 1. These are typical of the

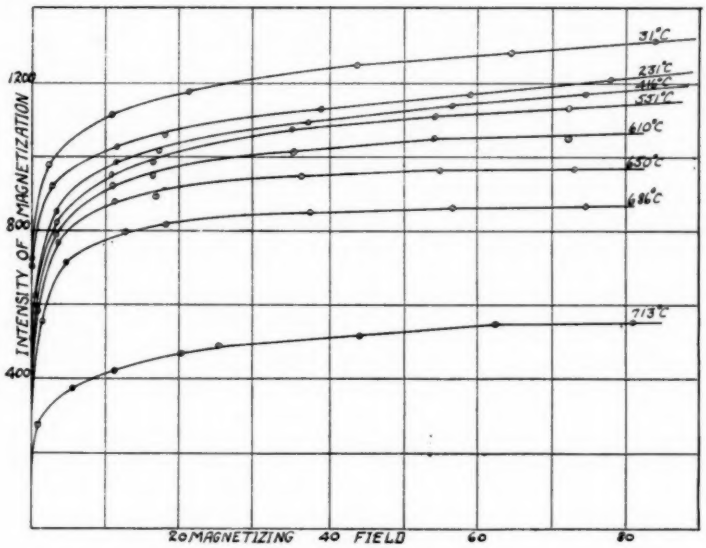


Fig. 1.

isothermals obtained for all five steels. The abnormally high susceptibility for low magnetizing fields, which seems to be the most pronounced characteristic of anhysteretic curves, persists up to the immediate neighborhood of the transformation point. The curves are uniformly concave to the $H$-axis and do not intersect each other except at the origin, though the curves are not conclusive on this latter point. The true magnetizing field is obtained as the difference between the applied field and the demagnetizing field due to the ellipsoid itself and this results in relatively great uncertainty in the value of $H$ when the field is weak. In the case of the softest of the steels, this difference was less than the error in the observations for applied fields as large as ten gauss and intensities of magnetization as high as nine hundred.

If, as seems likely, any given isothermal lies wholly beneath any other which corresponds to a lower temperature than the former, the magnetization for a given field decreases with a rise in the temperature. In the absence of hysteresis we do not find the anomaly common to ordinary magnetization curves, namely, that the magnetization for a given field

may either increase or decrease with rise of temperature, depending upon whether the field is weak or strong. In Fig. 2 are found the magnetization curves for all five specimens for a constant field of sixty gauss. It is evident that the curve for specimen *IN1*, which is most nearly free from carbon, is by far the most regular. The effect of carbon is to produce two irregularities, the one just above 700° C. and the other in the neighborhood of 200° C. The former corresponds to the precipitation of the carbides which occurs at the eutectoid point. The latter is due to the magnetic transformation of the cementite in the steel. The actual shape of the curves at this lower transformation point is not definitely indicated by the data but must be somewhat as shown by the dotted portions. The transformation point certainly lies between 180° C. and 220° C. which agrees with Honda's[1] work on cementite. For the purpose of the present investigation, a more exact knowledge of the curves in this region was not necessary.

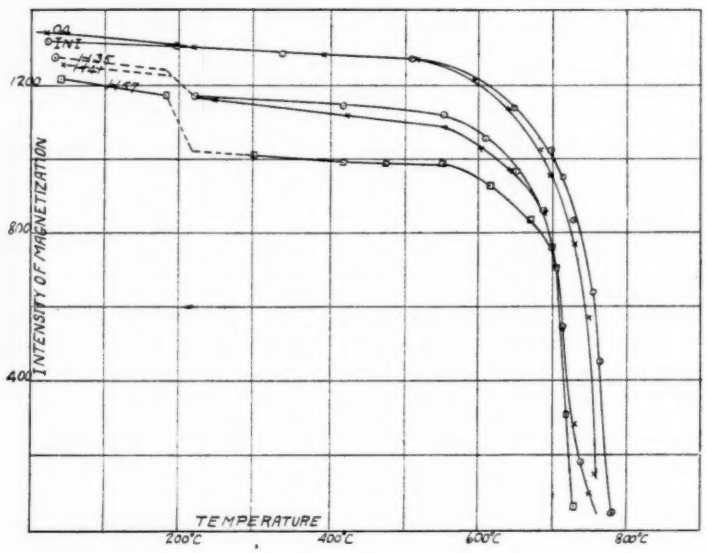


Fig. 2.

## Equations for Anhysteretic Curves.

Examination of the curves in Fig. 1 shows them to be smooth and regular, whether they are for iron or steel. The curves have at least three distinguishing characteristics, a uniform concavity to the *H*-axis, an infinite slope at the origin, and a finite limit to the ordinate as the abscissa increases indefinitely. The curves, of course, furnish no con-

[1] Honda and Takagi, Journ. Iron and Steel Inst., 92, 181, 1915.

clusive proof of the third characteristic but they indicate nothing contrary to this, the ordinarily accepted view. The appearance of the isothermals suggests the possibility of obtaining an equation for a given curve. But Fig. 2 shows clearly that to introduce the temperature as a variable in the equations will be feasible only in the case of carbon-free iron.

The only equation yet proposed for anhysteretic isothermals is empirical and due to Fröhlich. Finzi and Ashworth (loc. cit.) have both attempted to apply this equation to their experimental results. Steinmetz[1] has shown that the same equation may be fitted to limited ranges of the ordinary magnetization curve and expresses his opinion that it should probably fit an anhysteretic curve throughout its whole extent. The equation is based upon the assumption that the susceptibility is proportional to the amount by which the magnetization may yet be increased. Expressed in symbols, this becomes

$$\frac{I}{H} = K(I_0 - I),$$

where $I_0$ is the maximum intensity of magnetization and $K$ a constant. This may be transformed into the more useful form

$$H\left(\frac{1}{I} - \frac{1}{I_0}\right) = A,$$

where $A$ is a new constant. This equation is hyperbolic in $H$ and $I$, but is linear in $H$ and $H/I$. In Fig. 3, the data of Fig. 1 are plotted with this second pair of variables as coördinates and it may be seen to what extent the linear relation holds. Where a straight line fails to fit the points, a continuation, either straight or curved, has been made which will do so, in order that there may be less confusion as to corresponding lines and points. The continuations have been indicated by the dotted lines. In none of the isothermals does the equation seem to hold for fields much less than twenty gauss and, in two, at least, there is an indication that the relation is not linear through the upper range of available fields. This failure of the equation to hold may be considered from another viewpoint. If we form the derivative from the equation, we obtain

$$\frac{dI}{dH} = \frac{1}{A}\left(1 - \frac{I}{I_0}\right)^2.$$

[1] Steinmetz, Theory and Calculation of Electric Circuits, p. 54.

At the origin, this becomes

$$\left(\frac{dI}{dH}\right)_0 = \frac{1}{A}$$

and the condition for infinite slope can be fulfilled only by making $A$ vanish, which would reduce the hyperbola to two straight lines.

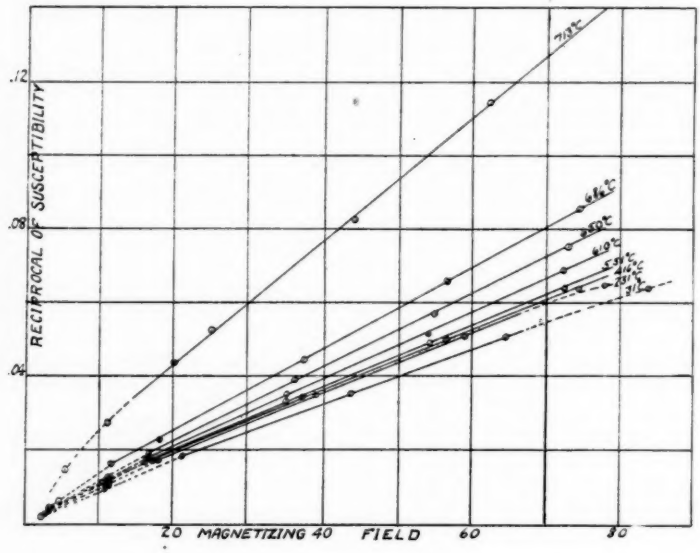


Fig. 3.

The results indicate that Fröhlich's equation does not fit the anhysteretic isothermal magnetization curves, though it may be made to fit a limited range of any curve and may be used as an approximation for the curve. Ashworth[1] not only accepts Fröhlich's equation for a given isothermal but attempts to use it for the whole family by introducing the temperature as a third variable. He does this from analogy with Van der Waal's equation and writes Fröhlich's equation in the form

$$H\left(\frac{1}{I} - \frac{1}{I_0}\right) = RT,$$

where $R$ is a constant and $T$ the absolute temperature. For $H$ constant, this equation is hyperbolic in $T$ and $I$. But the hyperbola of the equation is convex toward the $T$-axis whereas the curves of Fig. 3 are concave. This difficulty might be met by assuming that $I_0$ is a function of the temperature but, since it is an unknown function, it does not seem that Ashworth's equation is a step in advance of that of Fröhlich.

[1] Ashworth, loc. cit.; also Phil. Mag., 33, 349, 1917.

SUMMARY.

1. A null method for using the magnetometer has been described.

2. From a series of steels with decreasing carbon content, the anhysteretic magnetization curves for iron have been approximated and certain characteristics of these curves have been pointed out.

3. It has been shown that the equation proposed by Fröhlich does not fit the anhysteretic isothermal magnetization curves and that the equation, even when modified as Ashworth proposes, does not give the magnetization properly related to the temperature.

In conclusion, the writer wishes to acknowledge his indebtedness to the late Professor K. E. Guthe, at whose suggestion the work was undertaken, and to the members of the department of physics of the University of Michigan.

UNIVERSITY OF MICHIGAN,
May, 1917.

# THE IONIZATION POTENTIAL OF MERCURY VAPOR AND THE PRODUCTION OF THE COMPLETE SPECTRUM OF THIS ELEMENT.

BY T. C. HEBB.

THE results of Davis and Goucher[1] and of Bishop[2] seem to prove the correctness of the suggestion of Van der Bijl[3] that the apparent ionization of mercury vapor when bombarded with electrons possessing a velocity of 4.9 volts, as observed by Franck and Hertz[4] and by Newman,[5] was due to the photo-electric action of the radiation $\lambda = 2536.7$ acting on the receiving plate of the ionization chamber. Their results are also in harmony with those obtained by McLennan and Henderson[6] and also by Tate,[7] viz., that the ionization potential of mercury vapor is 10.3 volts.

Neither the above suggestion nor the experimental results quoted, however, explain the results obtained by the writer:[8] viz., that the complete spectrum of mercury vapor appeared at 4.9 volts, and that an arc struck at that voltage. Millikan,[9] however, has suggested that the same radiation $\lambda = 2536.7$ acts photo-electrically on the mercury vapor and produces the necessary ionization. If this should prove to be true then the fact will cast some light on the photo-electric action.

If, however, it should be found that photo-electric action is not sufficient to explain the arc at 4.9 volts, it would seem to be necessary to assume that under certain conditions mercury vapor can be ionized by collision with electrons moving with a velocity of 4.9 volts.

The experiments reported in this paper were undertaken in the hope that the above question might be decided. The results obtained will be dealt with under the following five heads: (1) Arcing voltages; (2) Current-potential Relations; (3) Striations; (4) Ionization Potential and (5) Photo-electric Action.

[1] Davis and Goucher, PHYS. REV., Vol. 10, p. 101, Aug., 1917.
[2] Bishop, PHYS. REV., Vol. 10, p. 244, Sept., 1917.
[3] Van der Bijl, PHYS. REV., Vol. 9, p. 173, Feb., 1917.
[4] Franck and Hertz, Deutsch. Phys. Gessell. Verh., Vol. 11, p. 512, 1914.
[5] Newman, Phil. Mag., Vol. 28, p. 753, Nov., 1914.
[6] McLennan and Henderson, Proc. Roy. Soc., *A*, Vol. 91, 1915.
[7] Tate, PHYS. REV., Vol. 7, p. 686, June, 1916.
[8] Hebb, PHYS. REV., Vcl. 9, p. 371, May, 1917.
[9] Millikan, PHYS. REV., Vol. 9, p. 378, May, 1917.

## 1. ARCING VOLTAGES.

The apparatus employed was essentially the same as that used in the previous work on the mercury arc. A horizontal section is shown in Fig. 1. *A* was a glass tube about 20 cm. long and about 2.5 cm. in diamteter. *B* and *C* were iron caps which were fastened to the tube with Khotinsky cement. Through *B* and *C* passed the iron electrodes which carried the anode *D* and the cathode *E*. Pump connections were made at both *F* and *G*, so that in case there was a small leak the resultant air did not have to pass across the arc space *DE*. All the joints were sealed

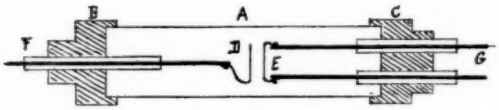


Fig. 1.

with Khotinsky cement. The anode *D* was of platinum foil usually about one centimeter square. The Wehnelt cathode was also of platinum foil .003 cm. thick and about 1.0 cm. in length. Its width was usually about .4 cm. The current used to heat the cathode varied between 10 and 20 amperes. Directly under *D* and *E* the glass tube was expanded into a depression in order to hold the mercury and the expansion was graded from the two ends of the tube, so that as fast as the mercury condensed at the ends it ran back. This kept a constant supply of mercury under the arc DE. The two iron caps, *B* and *C*, were surrounded by cooling vessels through which water circulated. The central part of the tube was surrounded by a gas-heated asbestos furnace with a sheet-iron bottom. The mercury evaporated at the center of the tube and passed both ways to the ends where, as stated above, it was condensed and returned to the center. As a result there should be produced in the region *DE* an atmosphere of the purest mercury vapor. This should be true even though the vacuum produced by the pump was not very high. As a matter of fact the pump used gave a minimum pressure of .25 mm.

With the apparatus as outlined above it was possible for me to substantiate my previous result, viz., that the arc could be caused to strike at a potential difference as low as 4.9 volts. But in doing so I found that it could be caused to strike at any potential difference above 4.9 volts by varying some or all of the following factors: (1) The temperature of the cathode, (2) the temperature of the furnace, (3) the distance between the anode and cathode and (4) the pressure as recorded by a McLeod gauge.

Curve *A*, Fig. 2, gives the relation between the striking voltage and the amperes through the cathode for a particular case. The McLeod gauge reading was 1.9 mm. In considering this curve it should be noted that the distance between the mercury and the cathode was about one centimeter and hence a rise in temperature of the cathode caused a more rapid evaporation of the mercury. That this made a difference was proved by the observation that the slope of the curve decreased when the mercury was not directly under the anode and cathode.

In regard to the effect of the second factor mentioned above, it may be stated that the striking voltage decreases with a rise in the temperature of the furnace, although I have no exact data to offer.

And in regard to the third factor I found that the striking voltage increased with the distance between the anode and cathode.

Curve *B*, Fig. 2, shows the relation between the striking voltage and the reading of the McLeod gauge for a particular case. Everything else was kept as constant as possible. It will be noticed that the striking voltage decreases with the pressure, reaches a minimum and then rises again. No significance should be attached to the fact that the curve starts at about 10 volts and rises again to that value. It could have been extended and was in some cases. The minimum point of the curve only reaches the value of 6 volts, but curves could be obtained in which the minimum potential difference had any value above 5 volts.

As a result of my experiments on arcing voltages I have come to the conclusion that the striking of the arc at 4.9 volts depends on (1) the density of the electron stream, (2) the density of the mercury vapor, and (3) the purity of the mercury vapor. If the electron discharge is weak or if the density of the mercury vapor is low, then there will be no arc formed at these low voltages. Further if there is the slightest trace of a foreign gas present, then, even though other conditions are favorable, the arc will not strike as low as 4.9 volts. This last condition is as would be expected if one considers the path of an electron which leaves the cathode and moves towards the anode through a dense atmosphere of mercury vapor. Owing to the elasticity of the collisions between electrons moving with speeds of less than 4.9 volts and molecules of mercury vapor, the electron probably makes many excursions back and forth past a certain point before passing on to the anode. If there were a molecule of an inelastic gas at that point the probability of collision with this molecule and the consequent loss of the electron's energy would be great.

### 2. Current-Potential Relations.

Using the same apparatus with a low resistance galvanometer in series with the experimental tube I made a study of the current-potential

relations in the arc. With this arrangement I found, as would be expected from a consideration of the previous results, that I could get a current-potential curve which took a decided bend at about 5 volts. I also found that by varying the same conditions previously mentioned under Arcing Voltages I could get the bend to occur at any potential

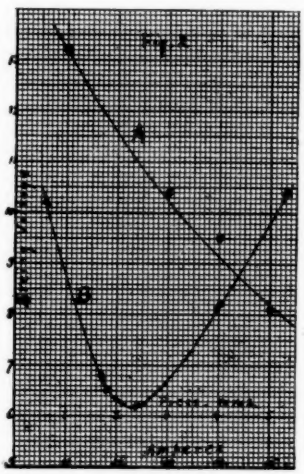

Fig. 2.

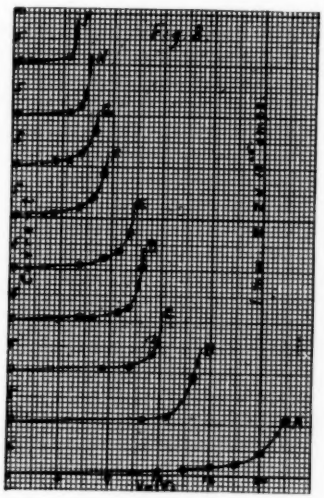

Fig. 3.

difference greater than 5 volts. Fig. 3 shows nine current-potential curves taken for a certain arrangement of the tube. The same cathode was used in all cases with the same current of 20 amperes flowing through it. The distance between the anode and cathode was about 3 mm. In all cases the rapid rise in current led to the striking of the arc. Most of these values were too large to represent on the diagram but they were utilized in getting the shape of the curve. Curves *A*, *B*, *C*, *D* and *E* were produced at pressures of 10, 6, 4.5, 2.2 and 1.3 mm. respectively, as recorded by the McLeod gauge. Everything else was kept constant, but the temperature of the furnace was low. Curves *F* and *G* were produced at the same pressure of .35 mm., but the temperature of the furnace was higher than in the previous cases. In the case of *G* the evaporation of the mercury was more rapid than in the case of *F*. For curve *H* the pressure was 2.7 mm. and the temperature of the furnace was still higher. Curve *I* was produced at a pressure of 5 mm. and at a continued high temperature. I did not determine these curves for the purpose of representing them together and hence the differences between the conditions under which they were taken are quite erratic.

### 3. STRIATIONS.

The conditions in a vacuum tube with a Wehnelt cathode are favorable to the study of striations, and as the striations radiate the complete spectrum of the element they appear to afford a method of studying the minimum voltage necessary to produce this radiation. In order to prevent the arc from striking, however, it is necessary to work with a comparatively cool cathode. I found that a red-hot cathode separated about 5 mm. from the anode gave very satisfactory results when the pressure was from 1 to 3 mm. If, with these conditions and with the temperature of the furnace low, the potential difference between the anode and cathode was raised, light appeared on the surface of the anode. The potential difference at which this occurred was never low, but usually in the neighborhood of 10 to 12 volts. If then the voltage was still further raised, the light on the anode grew towards the cathode and a portion of it separated from the main body of light on the anode. In all cases I found that the increase in potential difference necessary to produce this separation was 5 volts. If after the first striation was formed the potential difference was further increased the phenomenon repeated itself, the first striation in the meantime having moved nearer the cathode. The formation of this second striation also required the addition of 5 volts. The production of each new striation required an extra 5 volts. I have had as high as four distinct striations and the light on the anode with a potential difference of 32 volts. In this case the initial light was produced at 12 volts. On the other hand, with the furnace at a high temperature I have had two striations and the light on the anode for a potential difference of 15 volts.

The difference between the case where the temperature of the furnace is low and the case where it is high probably lies in the purity of the vapor between the anode and cathode. If the temperature is low, then the evaporation of the mercury will not be sufficient to drive away all foreign gases and as a consequence the electrons will lose energy in passing from the cathode to the anode. As a consequence a potential difference greater than 5 volts is required to produce light on the anode. In spite of this loss, however, the addition of 5 volts will make a second ionization possible. In the case where the furnace was at a high temperature, however, the evaporation of the mercury was sufficiently rapid to drive away all foreign gases and as a consequence the electrons lost no energy, other than that due to ionization, in passing from the cathode to the anode.

In connection with striations it may be of interest to state that I have had them so close together and so close to the cathode that I could only see the faintest dark line separating them.

## 4. IONIZING POTENTIAL.

It was considered both of interest and of value to determine the ionizing potential of mercury vapor under conditions somewhat similar to those in the preceding experiments, that is, to determine the ionizing potential in an absolutely pure atmosphere of mercury vapor having a pressure of one or more millimeters. In order to make this determination the anode of Fig. 1 was replaced by an ionizing chamber. The chamber consisted of a platinum cylinder about 4 cm. in length and 1.5 cm. in diameter. The end of the cylinder near the cathode was covered with platinum foil containing three slits. The central slit was about 1 cm. in length and about .3 cm. in width. The others were somewhat smaller. A small receiving disk was placed about 2 cm. from the cathode end of the cylinder. The cathode was separated 1–3 mm. from the end of the cylinder. Mercury stood under both cathode and ionization chamber and the apparatus was heated as usual. The ionization chamber was kept charged to a constant positive potential of 24 volts. The cathode was charged to a positive potential of less than 24 volts and hence the electrons were accelerated as long as they were between the cathode and ionization chamber. As soon, however, as they got inside of the chamber they were retarded.

The gold-leaf electroscope was set to a sensitiveness of about .05 volt per division. There was a condenser in parallel with it and the two together—condenser and instrument—had a capacity of about 230 e.s.u. With it, therefore, it was possible for me to measure currents as large as $10^{-9}$ amperes, when charging it to 5 volts. I was not able, however, to measure very small currents accurately, for I found that the passage of the mercury vapor over the receiving disk charged it positively. This was reduced to a minimum by arranging the apparatus so that the receiving disk was near the center of the furnace. But even under these conditions many observations were vitiated, apparently, by a sudden rush of vapor.

The results plotted in curve $A'$, Fig. 4, were taken without the use of the capacity mentioned above. The McLeod gauge registered 1.2 mm. The distance between the anode and cathode was 1.0 mm. The cathode was new and uniformly coated with BaO and was heated by a current of 19.5 amperes. The current flowing between the anode and cathode varied from $14 \times 10^{-5}$ amperes at 4.5 volts to $23 \times 10^{-5}$ amperes at 5.3 volts. The minimum potential difference between anode and cathode has been used as abscissa. It is quite evident that ionization must have started at about 4 volts, and as the drop in potential along the cathode was .8 volt the ionization potential must have been in the vicinity of 4.8 volts.

The results obtained with an old cathode are sometimes very marked due to the fact that the BaO wears down to narrow patches at either end of the cathode. Curve $B'$, Fig. 4, was plotted from data taken with such a cathode. The McLeod gauge reading was .85 mm. It will be noticed that the curve is very steep. As a matter of fact the ionization current increased over one hundred times when the potential difference between the anode and cathode was changed from 3.9 to 4.0 volts. It was possible to estimate quite closely the potential drop in the cathode at this point and this value—.9 volt—added to 4.0 volts gives 4.9 volts.

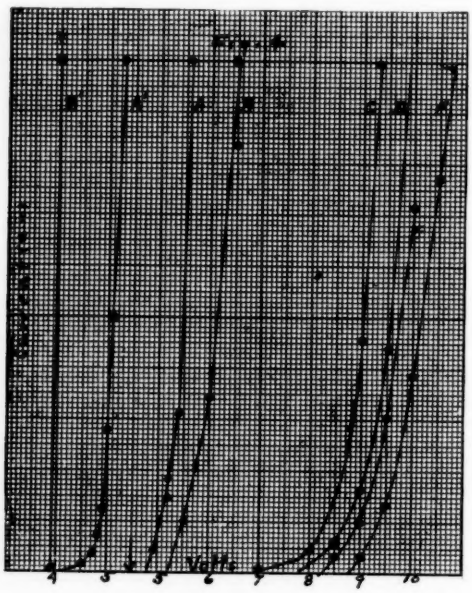

Fig. 4.

The curves shown—$A'$ and $B'$, Fig. 4—are similar to those obtained by other experimenters but I do not believe that the results can be explained on the assumption that the radiation $\lambda = 2536.7$ has acted photo-electrically on the receiving plate of the ionization chamber. Although such an action must have existed, the current produced by such action in these experiments must have been very small compared with the currents measured. This was especially true as the receiving plate had an area of only .25 square centimeter. And even when the receiving plate consisted of a small platinum wire sealed in glass it was found that the ionization current was still large.

Although ionization of mercury vapor occurred at 4.9 volts under favorable conditions, as shown above, it was also possible to get it to occur at any potential difference above 4.9 volts. This was accomplished

by simply varying the temperature of the furnace. Some results are shown in Fig. 4—Curves *A–F*. The data for these curves were taken with a constant current of 16.5 amperes through the cathode. The latter was separated about 1.5 mm. from the anode. The pressure indicated by the McLeod gauge was constant at .8 mm. The temperature of the furnace, however, was progressively lower, beginning with curve *A*. The gap between *B* and *C* could have been filled with similar curves had it been desired. The data for the curves *C–F* were taken as the temperature of the furnace was gradually lowering. This accounts to a great extent for the tendency of the curves to bend to the left as they approach the P.D. axis. The abruptness with which the curves drop into the P.D. axis is very pronounced in some cases. For example, in one case the ionization current at 10.8 volts was too small to be detected, if it existed at all. When, however, the potential difference was increased to 11 volts, the ionization current became about $10^{-9}$ amperes.

These results appear to me to prove that in order to get ionization of mercury vapor at 4.9 volts under conditions similar to those in my experiments it is necessary to have the vapor absolutely pure.

## 5. Photo-Electric Effect.

The previous experiments seem to prove conclusively that there is a distinct and pronounced ionization at 4.9 volts. But this ionization may be due to photo-electric action in a manner suggested by Millikan.[9] Further than that the results of Davis and Goucher[1] and also of Bishop[2] would appear to prove that such was the explanation. But even if some such action as Millikan suggests took place, it does not seem possible that the effect would be large enough to explain the results. It does not seem possible that radiation which required two or three hours to effect a photographic plate could produce $10^{-9}$ amperes photo-electrically as I have measured. Nor does it seem probable that the same radiation, even by the reciprocal action suggested by Millikan, could cause such large increases in the arc currents as I have obtained. For instance, in one case the current flowing between the anode and cathode changed from $10^{-5}$ amperes at 5 volts to $40 \times 10^{-5}$ amperes at 5.5 volts without the production of an arc. And in the following case where the arc struck the increase was much greater. In this case the current changed from $3 \times 10^{-5}$ amperes at 5 volts to $540 \times 10^{-5}$ amperes at 5.3 volts. Still another objection to the theory, it appears to me, is the fact that striations can be obtained in mercury vapor and especially in such close proximity to one another.

However, in order to test whether the photo-electric action was the cause of the ionization I arranged a mercury arc in air directly outside of the experimental tube and so arranged that its light passed into the front end of the ionization chamber. The experimental tube had been exchanged for one of quartz. Conditions were then arranged so that a large ionization current was produced by the electrons from the cathode. The voltage between the anode and cathode was then reduced to zero and the mercury arc in air started. It was found that the photo-electric current produced by the 4-ampere arc was, in some cases, equal to the current produced by the electron stream. Thus in one case at a pressure of 2.9 mm. and with 5 volts between the anode and cathode the electroscope charged up to 2.5 volts in 4 seconds. The voltage between the anode and cathode was then reduced to zero and the mercury arc in air started. The latter produced exactly the same ionization current. A carbon arc produced no results. As the effective radiation produced by a 4-ampere arc must be hundreds of times greater in intensity than the radiation $\lambda = 2536.7$ produced in the experimental tube by the electron discharge due to 5 volts, it does not seem probable from this result that the ionization produced by the 5 volts could have been due to the radiation $\lambda = 2536.7$.

It was further found that changing the pressure in the tube had very little effect on the ionization produced by the arc whereas the same changes caused the ionization current produced by 5 volts to vary from zero to a large value.

I also tried to find what effect the mercury arc in air had upon the striking voltage of the arc in the vacuum. Conditions were arranged so that the arc in the vacuum struck at 8 volts. The voltmeter was then

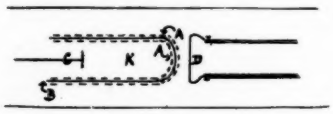


Fig. 5.

set at 7.9 volts and the mercury arc in air started. If it sets up ionization in the tube of sufficient amount, then one would expect the arc in the vacuum to strike lower than 8 volts. No such effect was observed.

The ionization chamber was then arranged as in Fig. 5.

Two platinum cylinders *AA* were separated by a quartz test tube *B* and together with *C* as a receiving plate constituted the ionization chamber. *A* and *A* were joined electrically and a potential difference of 24 volts was applied. No electrons from the cathode *D* could get into the ionization chamber *K*. Mercury was kept in the chamber *K* as well as under

the arc $DA$. The current flowing between the anode and cathode was measured and hence I could tell when ionization began. Some of the radiation produced should pass into the chamber $K$ and produce ionization. Of course some of the radiation was absorbed by the quartz test tube which was about 1 mm. in thickness and some was absorbed by the mercury vapor which formed an unavoidable layer between two cylinders $AA$. This layer was about .5 mm. in thickness. As mentioned before the passage of mercury vapor over the receiving plate $C$ causes it to be charged with positive electricity. I could not obviate it in this case as in the previously mentioned one and hence it was not possible for me to detect extremely small currents but in no case did I detect any current due to the radiation $\lambda = 2536.7$. That the apparatus would have responded as expected if the radiation $\lambda = 2536.7$ had produced ionization in sufficient quantity was proved by the fact that the slightest arc between $D$ and $A$ produced a rapid charging of the electroscope. My experiments along this line, therefore, have not shown, so far, any evidence of an ionization of mercury vapor by the ratiation $\lambda = 2536.7$.

## SUMMARY.

1. These experiments prove conclusively that mercury vapor may be ionized when bombarded with electrons moving with a velocity acquired in falling through 4.9 volts and that the complete spectrum of mercury is produced as a result.

2. Experimental evidence is given to show that this ionization is not produced by the radiation $\lambda = 2536.7$ acting photo-electrically on the mercury vapor.

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# ON EQUILIBRIUM UNDER NON-HYDROSTATIC STRESS.

BY P. W. BRIDGMAN.

IN a recent number of the PHYSICAL REVIEW[1] Williamson has published a paper on the subject of the above title, which was also the subject of an earlier paper of my own. Since Williamson's paper is partly in criticism of mine, and since the subject is growing to be one of important bearing in geophysics, it is perhaps appropriate that I should attempt to make clearer the point of view of my original paper. With regard to my paper Williamson says: "More recently Bridgman has deduced a formula of very formidable appearance dealing with the change of melting point and transition point with stress. He unfortunately also makes no mention of assumptions, giving as his reason for this: 'The formulas were derived by ordinary thermodynamic methods; it is hardly worth while to reproduce the wearisome details.' As regards the mathematical transformations this is true, but we hope to show that several of his terms rest on very shaky foundations." Later he says: "The fundamental assumption made is that of reversibility, which is a necessary premise to the equality of the potential ($\mu$). This assumption needs some explicit criticism and justification." It seems, therefore, that some comment is necessary on the method of deducing my formula, and on the question of reversibility.

The formula, as Williamson hints, is of considerable generality; this generality consists not only in the range of conditions of stress and crystalline structure covered by it, but also in the variety of assumptions with regard to the nature of the contact conditions which it makes possible to the user of the formula. I supposed that in general the two phases were separated by a membrane permeable to the phases, but such as to support a stress difference. The nature of the stress difference supported by the membrane may vary with conditions, and must be appropriately specified in each case. In particular, if there is no membrane at all, the forces on the two sides of the surface of separation are the same, and this case is also covered by the formula on making the appropriate substitutions. I did not at all intend to touch the question as to whether such membranes actually exist; my purpose in giving a formula of such generality was that many writers have supposed that in

[1] E. D. Williamson, PHYS. REV., 10, 275–283, 1917.

actual cases the action is of this nature. The assumption, however, as to the nature of the action must in each case be made by the user of the formula. It is significant that my own position in the actual cases discussed was that the conditions are not such as are represented by a membrane, but that the usual conditions of mechanical equilibrium must hold at the interface.

Thermodynamics is competent to state whether equilibrium exists or not under such conditions. I intended to make clear by my statement that "the formulas were derived by ordinary thermodynamic methods" what the nature of the assumptions was. Taking the existence of such membranes as a fact, the only assumption made in deducing the formulas was that the first and second laws of thermodynamics hold, and that the solid is always strained within its elastic limit. Here enters the second point requiring elucidation; it is objected that the laws of thermodynamics cannot be applied without making the assumption that the reaction is reversible, and that in special cases experiment is needed to justify the assumption of reversibility. This does not seem to me an accurate statement; we are in most cases given a great deal of information about the reversibility of an operation by thermodynamics only, without any further experiment.

Thermodynamic and mechanical reversibility must be clearly distinguished; the only condition demanded in such work as above is that there be thermodynamic reversibility. The simplest statement of thermodynamic reversibility is that the entropy increment in any change (at constant temperature) shall be $\Delta Q/T$, where $\Delta Q$ is the heat absorbed. To find the actual difference of entropy between two phases it is necessary that we pass from one to the other by some path known to be reversible. It would of course be begging the question to pass directly from one phase to the other and forcibly write down the entropy condition. Now the derivation of the formulas of my paper consisted essentially in passing from one phase to the other by a process incontestably reversible, and from this obtaining the conditions for a direct thermodynamically reversible change. The incontestably reversible method of passing from one phase to the other is by an elastic change of stress on one phase until the hydrostatic pressure of two-phase equilibrium at that temperature is reached, reversible change to the other phase under equilibrium conditions at the determinate hydrostatic pressure, and elastic change of stress on the second phase, bringing it to the required final conditions. This process constitutes three of the four parts of the complete cycle referred to on page 217 of my paper, and it is the information given by this part of the cycle that ensures that the fourth and closing step, direct

passage from one phase to the other, is thermodynamically reversible. The details are wearisome, it is true, but are perfectly straightforward, and were certainly not worth the expense of publication.

The argument thus far has established thermodynamic reversibility under the conditions specified in the formula. Physically this *may* mean equilibrium (equilibrium is here used in the sense of no spontaneous change), for (in general) neither phase can change to the other under the conditions imposed by the formula without the addition of energy to the system. More than this, thermodynamics says that if the system is displaced in a certain direction, a certain change *may* take place, and a certain other change *cannot* take place. As to whether any change at all takes place when the system is displaced, or whether equilibrium fails because an entirely different kind of change takes place, such as the appearance of another different phase, thermodynamics has absolutely nothing to say. This is a question of mechanical reversibility; thermodynamic equilibrium is a necessary but not a sufficient condition for it. The matter of mechanical reversibility can usually be settled only by direct appeal to experiment. Besides settling the question as to mechanical reversibility, experiment may often perform another useful function. In many cases it may not be physically obvious that the variables chosen to specify the state of the system are adequate. Thus in the example of the next paragraph, that of a crystal growing under stress, it might be feared that other variables, such as the surface tension and curvature of the surfaces of separation, might be needed to adequately specify all the physical factors. In such cases, experimental proof of mechanical reversibility under the conditions demanded by a thermodynamic discussion raises strong presumptive evidence that the variables assumed in the thermodynamic discussion afford a physically adequate description of the phenomena. It would make an interesting topic to find just how strong the presumptive evidence is in different cases.

In this connection it is pertinent to mention a recent paper by Wright and Hostetter, also from the Geophysical Laboratory. They have with great experimental skill examined crystals growing under stress, and have proved to their own satisfaction that the "assumption" of reversibility is justified. In view of the above, it seems to me that the point of their work is not exactly as they represent it. They have established experimentally the *mechanical* reversibility of the growth on the free face of a strained crystal. This is certainly an important contribution, most significant for our conception of the nature of the crystal building forces, and necessarily one obtainable only from experiment. But their results are entirely superfluous as far as *thermodynamic* reversibility goes;

if there is no blunder, the formula above is as secure as thermodynamics itself, and tells us without question that under certain conditions, one of which is the non-appearance of a new phase, and another of which is the physical adequacy of our assumed variables, the crystal can neither dissolve nor grow, and that under certain other conditions the crystal can only grow or only dissolve. Results such as this are always obtainable by thermodynamic reasoning, and the formula in question is no exception.

The only room left, it seems to me, for difference of opinion is with regard to applications of the formula; in any special case is the physical action correctly represented by a particular membrane or not? A case to be represented by a membrane is case *B* of Williamson, in which the pressure on the solid is not the same as that on the liquid in contact with it. One might possibly gather from his paper that this case plays a somewhat prominent part in mine; it was actually treated in just five lines (bottom of page 218) and was prefaced by the remark, "if we suppose the membrane such that, etc." I did not intend to argue whether in any actual case such a membrane exists, and I can, in fact, think of no case, except possibly an inert gas in contact with a liquid, in which I believe this adequately represents the facts, although other writers have. Williamson argues at some length that in any actual case, such as a weight resting on a solid, this does not adequately represent the state of affairs ,with which I agree, and deduces an expression which he prefers, which is the same as my case 5. I agree that this much more closely represents the state of affairs in any such case as the melting of snow under the runners of a sleigh. But Williamson finds an irreversible aspect in the lateral outflow of liquid, which to his mind makes the treatment uncertain; I am of the opinion that this irreversible aspect has no effect and that the formula accurately gives the conditions under which melting will just begin, because the irreversible process takes place only after melting has occurred, and cannot therefore affect the actual melting. The formula applies strictly only to equilibrium; any physical progress of the transition must imply, as always, a small element of irreversibility.

The substance of Williamson's criticism seems to me, therefore, to boil down to this; the formula should have been made much simpler by leaving out the membrane, because it is not certain that such membranes exist. This of course is a question of judgment. But that the formula, when correctly handled, gives correct results without assumptions which require further resort to experiment, of this there can be no doubt.

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# ELASTICITY OF IMPACT OF ELECTRONS WITH GAS MOLECULES.

BY J. M. BENADE AND K. T. COMPTON.

*Introduction.*—In the theory of ionization by collision, as originally developed by Townsend, it was assumed that an electron lost practically all of its kinetic energy at each impact with a gas molecule. Subsequent discoveries have shown that this view is substantially correct in the case of most gases.

On the other hand, it was pointed out by Franck and Hertz[1] that the strong ionization in helium, whose molecules are difficult to ionize, could only be explained by assuming that in this gas the electrons retained a considerable portion of their energy at encounters, so that their energy at any instant has been accumulated during the entire path since the preceding ionizing collision. They proved the existence of this type of collision in helium, and also in the other monatomic gases neon and mercury vapor, by showing that in these gases the ionization current increases abruptly whenever the applied difference of potential between the electrodes is increased to an exact multiple of the minimum ionizing potential.[2] These experiments have been amply verified by Goucher,[3] Bazzoni,[4] Todd and others. We are therefore justified in distinguishing two general types of impact, inelastic and elastic.

The question immediately suggests itself: "Are these really two distinct types of impact, or may there be all degrees of elasticity between the two extremes of perfect elasticity and complete inelasticity?" At first sight, the case of hydrogen appears to support the latter alternative. Impacts in hydrogen are known to be less elastic than those in monatomic gases, but more elastic than in other multiatomic gases.

As far as we know, the only attempts to measure directly the amount of energy lost by an electron at a collision were made by Franck and Hertz.[5] They projected electrons with a known maximum velocity

[1] Verh. d. D. Phys. Ges., 15, p. 34, 1913.

[2] Verh. d. D. Phys. Ges., 16, p. 457, 1914. Professor Bergen Davis and Mr. F. S. Goucher have shown, in the case of mercury vapor, that these successive discontinuities occur also at multiples of the "minimum radiating potentials."

[3] PHYS. REV., 8, p. 561, 1916.

[4] Phil. Mag., 32, p. 566, 1916.

[5] Verh. d. D. Phys. Ges., 15, p. 373, 1913.

through a gauze into a long chamber filled with gas at low pressure. If any electrons returned to an electrode in the plane of the gauze, these must have been reflected from gas molecules. By measuring the retarding field against which these reflected electrons could reach the electrode, the energy retained after a collision was found and thus the energy lost at a collision was determined. It is to be noted that, in these experiments, electrons approaching the detecting electrode obliquely were treated as if approaching directly, with the result that the apparent average energy loss was much greater than the actual loss. Realizing this, Franck and Hertz can only conclude that the average loss of energy at a collision, expressed in equivalent volts, is less than 0.3 volt in helium, 1.6 volts in hydrogen, and that in the common gases practically all the energy is lost. These considerations, of course, apply only to impacts with velocities less than the minimum ionizing velocity. When ionization occurs, the electron must lose at least the amount of energy necessary to ionize the molecule.

Recently the writers[1] have suggested a theory of the loss of energy by an electron while passing through a gas, according to which the electron should lose very little energy in a monatomic gas, whereas in multiatomic gases, the loss of energy should be least in light gases of simple molecular structure and greatest in heavy complex gases. Qualitatively, at least, this is in accordance with the facts. The vital point in the theory, however, is that *the loss of energy in the two types of gases is due to distinctly different processes*, so that we should *not* expect to find all degrees of elasticity of impact between the most and the least elastic gases.

In the present investigation we have developed a method for measuring the loss of energy at an impact which has enabled us to measure accurately losses of the order of magnitude of a thousandth of a volt. Measurements of the loss in helium indicate that impacts of electrons with helium atoms are perfectly elastic in their nature, *i. e.*, that the only energy lost by the electron is due to the motion imparted to the atom during impact. The method is so sensitive to changes in the elasticity of impact that the experimental measurements prove the coefficient of restitution at impacts in helium to be unity with a possible experimental error of not more than 0.01 per cent. In other words, the coefficient of restitution, if not unity, is at least greater than 0.9999. In hydrogen and oxygen the loss of energy is much greater and is shown to be of a more complicated type than in helium. Attempts to measure the loss in argon have failed, up to the present, owing to the failure to obtain gas of sufficient purity for these experiments. These points will

[1] PHYS. REV., 8, p. 449, 1916.

be discussed more fully after the experimental evidence has been presented.

*Calculation of the Average Energy lost by an Electron at a Collision with a Gas Molecule.*—Let us consider, for the moment, the case of an electron of mass $m$ moving with velocity $v$ and colliding in "head on" fashion with a stationary molecule of mass $M$. After impact the velocities of electron and molecule are $v_1$ and $V_1$ respectively. The electron loses a fraction $k'$ of its original kinetic energy, which we may easily calculate from the relations

$$mv = MV_1 - mv_1,$$

$$ev = V_1 + v_1,$$

where $e$ is the coefficient of restitution. We find

$$k' = \frac{v^2 - v_1^2}{v^2} = \frac{M^2(1 - e^2) + 2Mm(1 + e)}{(M + m)^2}.$$

Since we may take $M = M + m$ without appreciable error, this expression may be written

$$k' = (1 - e^2) + 2(1 + e)\frac{m}{M}.$$

In the actual case of electrons traveling through a gas, not all collisions are of the "head on" type, in which the velocities are in the direction of the line of centers at impact. Many electrons strike more or less "glancing" blows, and we have to average the effect of all. To make calculation possible we shall assume the molecule to be spherical. We may then multiply the energy lost by an electron which strikes the surface of the molecule at a given angle by the probability of striking at that angle, and integrate over all possible angles (0 to $\pi/2$), thus determining the average loss of energy at a collision. Even this calculation is difficult except in the particular case of interest to us, when $e$ is very near to unity. In this case we find approximately

$$k = \frac{(1 - e^2)}{2} + \frac{(1 + e)m}{M} \qquad (1)$$

for the average fraction of its energy lost by an electron at a collision. This approximation becomes more accurate as $e$ approaches unity, and if collisions are perfectly elastic the relation is exact, taking the form

$$k = \frac{2m}{M}, \qquad (2)$$

which is just half the value of $k'$ for "head on" collisions alone.

We have assumed, thus far, that the molecule is at rest when struck. The question therefore arises: What is the effect of the thermal motion of the molecules on the decrease in the kinetic energy of the electron at impact? We may take this into account by averaging the effects of two types of collisions: between electrons and molecules moving in opposite directions and between those moving in the same direction before impact. Assuming perfect elasticity and denoting the average molecular velocity before impact by $V$, we find that, out of $N$ collisions, there are

$$\tfrac{1}{2}N\frac{v+V}{v}$$

collisions of the first type, resulting in an average energy loss equal to

$$M\left\{\left(\frac{m}{M}\right)^2 v^2 + \frac{m}{M}vV\right\}$$

and

$$\tfrac{1}{2}N\frac{v-V}{v}$$

collisions of the second type with an average energy loss equal to

$$M\left\{\left(\frac{m}{M}\right)^2 v^2 - \frac{m}{M}vV\right\}.$$

We can therefore obtain the total loss of energy by the electrons in all $N$ collisions and thence find the average loss per collision. When this is divided by the average energy before collision, we obtain

$$k = 2\left(\frac{m}{M} - \frac{V^2}{v^2}\right). \tag{3}$$

This expression illustrates the equipartition theorem, for it shows that, in the absence of external forces, the two types of particles will exchange energy until their average kinetic energies are equal, when the proportion $k$ determining the average loss of energy at a collision becomes zero.

In the present case, however, the velocities $v$ with which we have to deal so far exceed the thermal velocity $V$ that the second term is entirely negligible in comparison with the first. We are therefore justified in taking equation (1) to represent the fraction of energy lost by an electron at a collision, if impacts are very elastic.

In the case of helium, substitution of the relative masses of an electron and a helium atom leads to the value

$$k = 0.0002685 \tag{4}$$

if the collisions are perfectly elastic.

We shall proceed to a description of the experimental method of determining $k$. If the experimental value of $k$ should differ somewhat from 0.0002685, the appropriate value of the coefficient of restitution could be calculated from equation (1).

*Method.*—Since the energy lost at a single collision between an electron and a monatomic molecule is known (or assumed for the present) to be small, in order to measure this loss it is necessary to deal with the aggregate effect of a large number of successive collisions. This has been done by liberating electrons at a negative plate and driving them through the gas to a second electrode parallel to the first and positively charged. The number of collisions made by an electron is a function of the gas pressure $p$ and the distance $d$ between the plates.

Curves representing the increase in the electronic current with increasing potential difference indicate by an upward inflection, or "break," the potential at which ionization begins; and this occurs as soon as an appreciable number of electrons have a quantity of energy, in the case of helium, corresponding to a fall through 20 volts, the well-known ionization potential. The difference between the applied potential and 20 volts represents the energy lost by collisions with molecules, and can be made as large as we please by increasing the product $pd$.

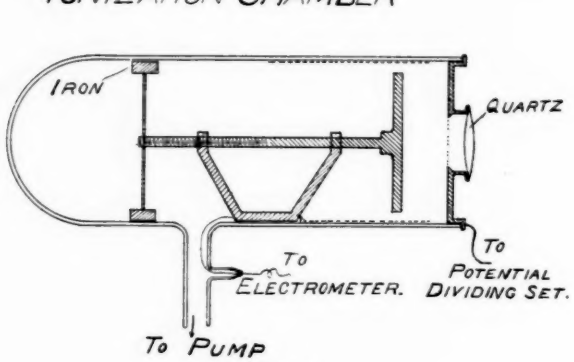


Fig. 1.

The earlier curves were plotted from data obtained by the use of an ionization chamber similar to that used by Partzsch in his work on Stoletow's constant,[1] but later a simpler and more satisfactory one was substituted. The latter is shown in Fig. 1. It consists of a glass tube of about 5 cm. diameter and 14 cm. length, with other parts as shown in proportion. The brass cap on the end, with a fine wire gauze flush with its inner surface serves as one of the electrodes. Behind this electrode

[1] Ann. d. Phys., 40, p. 157, 1913.

and parallel to it is a brass disk, the second electrode, which is mounted on a horizontal threaded shaft held in a nut and journal coaxial with the glass container. On the rear end of the shaft is a cross bar with two iron lugs which, with the aid of an external electromagnet, serve to adjust the distance between the electrodes. A wire gauze closely fitting the inner surface of the chamber surrounds the adjustable electrode and is electrically connected to it. This prevents the accumulation of a charge on the surface of the glass when the distance $d$ is large. The surfaces of both electrodes were heavily coated with platinum, by sputtering, to insure constancy of photoelectric effect and to avoid contact difference of potential.

Ultra-violet light from a quartz mercury vapor lamp enters the chamber through a quartz window and the gauze and liberates electrons from the movable electrode, which is connected to a sensitive quadrant electrometer shunted with a resistance of about 100 megohms. The electrometer gave a deflection of about 2,000 mm. per volt, so that the arrangement is equivalent to a galvanometer with a sensitivity of about $5(10)^{-12}$ amperes per division. (The shunt resistance was very satisfactory and consists of a thin film of platinum deposited on hard rubber or glass, with globules of mercury for contacts.) The fixed electrode is connected to a conveniently adjustable potential source, and voltmeter.

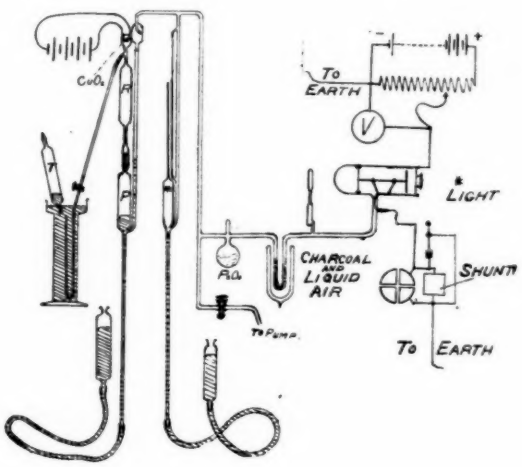


Fig. 2.

The ionization chamber is connected to a hand mercury pump, gas reservoir and McLeod gauge, as shown in Fig. 2. Before introducing gas for investigation, the apparatus was exhausted by a Gaede pump, allowed to stand for some time and again pumped down to the lowest

attainable pressure to get rid of adsorbed gases. As far as possible, the glass parts were heated during part of this process.

The helium was introduced as follows: A U tube, with one arm drawn out to form a capillary tube with the end open, was immersed in mercury as shown at the left of the figure. The stopcock above the U tube was opened while the apparatus was being exhausted and mercury allowed to rise in the tube to a point a little above the stopcock, so that the U tube was entirely filled with mercury and the open end was beneath the surface of the mercury in the cylinder. The tube containing the gas to be introduced was scratched and the end broken off under mercury. The end of the capillary was then introduced into this gas container, which was pressed down allowing the gas to be forced into the apparatus when the stopcock was opened. In this way not more than a cubic millimeter of gas was lost in the transfer. The first bit of gas transferred was pumped out again, in order to carry out traces of other gases remaining in the apparatus. Finally, the introducing tube was sealed off. By means of the hand pump the gas could be pumped from the ionization chamber into the reservoir, so as to get any desired pressure in the chamber. The mercury sealed valve between the pump and the reservoir carried an iron weight on the stem so that the valve could be held open by an electromagnet when it was desired to let gas flow back into the ionization chamber.

A spectrum tube connected with the ionization chamber was used with a direct reading Hilger spectrometer to indicate the presence of impurities in the gas. When working with helium, a U tube filled with cocoanut charcoal and surrounded by liquid air was used to remove impurities. This was very effective except in the removal of hydrogen. It was found that the hydrogen spectrum was much reduced when an electrodeless discharge tube was substituted for the original one, which had aluminium electrodes. This indicates that much of the hydrogen came from the electrodes, as had been proved by Winchester.[1] In order to remove the remainder of the hydrogen, the following method was found the most satisfactory of several methods tried. A small bulb containing a platinum coil which was heavily copper-plated and well oxidized was attached as shown in Fig. 2. After keeping the copper oxide at a bright red heat for several days the hydrogen spectrum was so much reduced as to be almost invisible at low pressure discharges, though it was quite evident at the higher pressures. In this connection it should be remembered that the presence of helium in a discharge tube has the effect of greatly enhancing the spectra of any other gases which

[1] PHYS. REV., 3, p. 287, 1914.

may happen to be present in the tube. It is possible, also, that most of the hydrogen observed was liberated by the discharge in the spectrum tube, and may not have been present in the ionization chamber during the tests. At any rate we feel quite safe in assuming that our helium could be considered pure, for a simple calculation shows that the results of our experiments would have been impossible had there been present in the gas as much as one part of hydrogen in one hundred thousand parts of helium. Several mercury lines were also faintly visible, but with the liquid air trap the amount of mercury vapor present could not have been serious, and even this small amount would not be likely to affect the results because it is fairly well established that collisions in mercury are elastic.

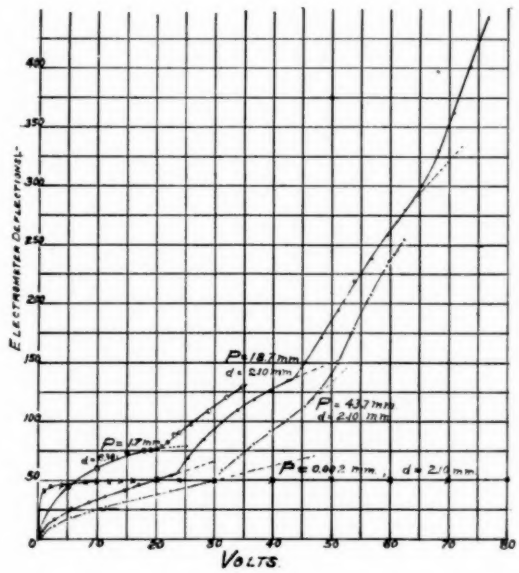


Fig. 3.

Figs. 3 and 4 give typical examples of a large number of curves obtained by plotting the electronic current in helium as a function of the applied potential $V$ for various gas pressures $p$ and distances $d$ between the plates. At extremely low pressures there is no evidence of ionization of the gas, the currents quickly reaching saturation as the potential drop is increased. When the product of the pressure and distance $pd$ is larger, so that an appreciable number of collisions occur, ionization sets in when the applied potential is 20 volts, as indicated by the "break" in the curve. For larger values of $pd$ this "break" is shifted toward larger values of the applied potential, proving that energy is being lost

by the electrons at collisions. Some of the curves are extended to show two or three "breaks," indicating potentials at which the electrons liberated by the preceding ionization are themselves ionizing the gas.

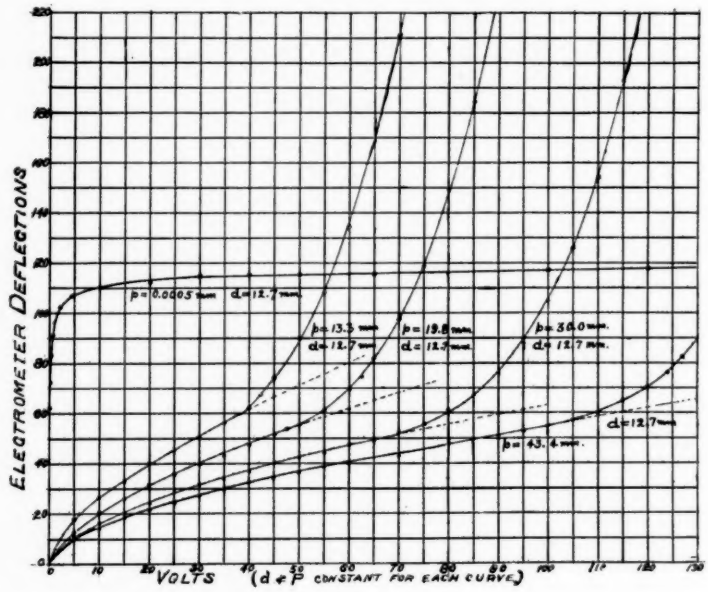
ELECTROMETER DEFLECTIONS
VOLTS (d ∝ P CONSTANT FOR EACH CURVE)
p = 0.0005 mm d = 12.7 mm
p = 13.3 mm d = 12.7 mm
p = 19.8 mm d = 12.7 mm
p = 36.0 mm d = 12.7 mm
p = 43.4 mm
d = 12.7 mm

Fig. 4.

It is interesting to note that the second and third "breaks" do not come at exactly two and three times the potential of the first, except when this is at 20 volts. This is due to the fact that the average number of collisions made by an electron while acquiring sufficient energy to ionize

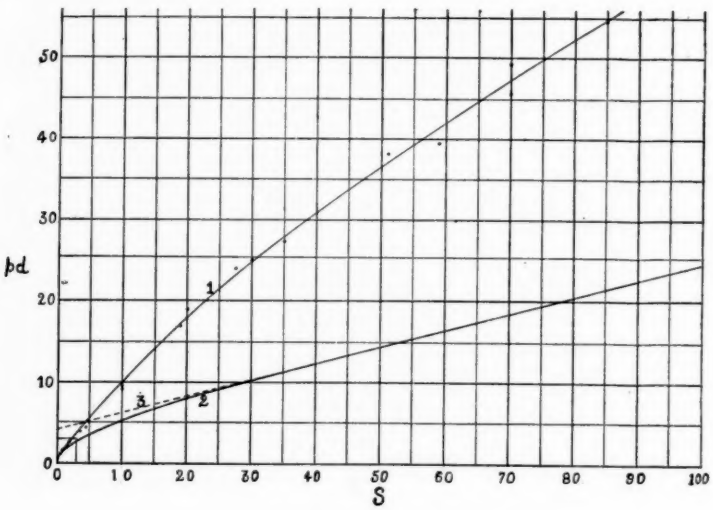


Fig. 5.

is less in the second and third cases than in the first because of the greater potential gradient in these cases.

The conclusions regarding the energy lost at impact are drawn from the variation of the shift of the "break" point (observed ionizing potential) with the product of the pressure and distance *pd*. The data from observations made after the apparatus was working satisfactorily are given in Table I. and are shown graphically by Curve 1, Fig. 5.

TABLE I.

| *p* (Mm.). | *d* (Cm.). | *pd*. | Shift *s*. |
|---|---|---|---|
| 11.70 | 0.425 | 4.97 | 3.7 |
| 11.70 | 0.2125 | 2.48 | 1.8 |
| 5.55 | 0.2125 | 1.18 | 0.5 |
| 4.35 | 0.900 | 3.91 | 3.7 |
| 4.35 | 1.010 | 4.39 | 4.5 |
| 4.34 | 0.476 | 2.06 | 1.6 |
| 4.34 | 0.370 | 1.60 | 0.8 |
| 4.34 | 0.265 | 1.15 | 0.5 |
| 46.60 | 0.846 | 39.40 | 59.0 |
| 46.60 | 1.060 | 49.30 | 70.0 |
| 43.00 | 0.636 | 27.30 | 35.0 |
| 43.00 | 0.848 | 36.50 | 60.0 |
| 43.00 | 1.060 | 45.50 | 70.0 |
| 43.00 | 1.270 | 54.60 | 85.0 |
| 43.40 | 1.270 | 55.00 | 85.0 |
| 30.00 | 1.270 | 38.10 | 51.0 |
| 19.80 | 1.270 | 25.06 | 30.0 |
| 13.30 | 1.270 | 16.90 | 19.0 |
| 43.70 | 0.210 | 9.20 | 10.0 |
| 18.70 | 0.210 | 3.93 | 3.7 |
| 10.55 | 1.800 | 19.00 | 20.0 |
| 13.30 | 1.800 | 23.95 | 27.5 |
| 5.40 | 1.800 | 9.72 | 10.0 |

In order to use these experimental results to determine the degree of elasticity of impact, it is necessary to picture to ourselves the phenomena accompanying the passage of an electron between the electrodes in the gas, and to express the energy of the electron at any point in its path in terms of the gain from the field and the loss from collisions.

*Change of Kinetic Energy of an Electron passing through a Gas.*—The photoelectric relation between the nature of the emitting cathode and the effective wave-length of the ultra-violet light is such that we may neglect the initial velocities of the electrons. We have to deal, therefore, with a group of electrons which start from rest at the cathode and move toward the anode, bounding and rebounding from the molecules with

which they collide. During each free path the motion of an electron is determined by the electric field and the velocity retained after its preceding impact. At each collision, however, a fraction $k$ of its energy is lost. Our problem is to determine the average energy $Ue$ of an electron after it has moved a distance $d$ through the gas at pressure $p$ under the action of a uniform field of intensity $X$, and thus to calculate the difference of potential through which the electron must move in order to acquire the energy necessary to ionize a molecule.

Let us express the average energy of an electron at any point in the gas by $Ue$, where $U$ is the energy in equivalent volts. The average rate at which the electron is acquiring energy at this point of its path is $e(dU/dx)$. However the electron gains energy from the field at the rate $eX$ per centimeter. Thus $e[X - (dU/dx)]$ represents the average amount of energy lost at collisions per centimeter advance toward the anode.

If $N$ is the average number of collisions made by an electron in a centimeter path through the gas at one millimeter pressure, then $pN$ is the average number of collisions per centimeter path at the pressure $p$. The average number of collisions made while *advancing* one centimeter toward the anode we shall denote by $\nu$, which is much greater than $pN$ because of the zig-zag character of the path. The relation between $\nu$ and $pN$ is found as follows:

During a free path $l$ the electron experiences an acceleration $X(e/m)$ in the direction of the field for a time equal to $(l/v)$, where $v$ is the average speed. Therefore

$$s = \tfrac{1}{2}at^2 = \frac{1}{2}\frac{Xel^2}{mv^2}$$

gives the average distance moved in the direction of the electric field during one free path. The reciprocal of $s$ is $\nu$ and of $l$ is $pN$, whence

$$\nu = \frac{2mv^2p^2N^2}{Xe}.$$

We could put $mv^2 = 2Ue$, were it not for the fact, discussed later, that the electrons quickly acquire Maxwell's distribution of speeds about the mean speed of advance, so that we must distinguish between the square of the mean speed $v^2$ and the mean square speed $\overline{v^2}$. Let the ratio $v^2/\overline{v^2}$ equal $r^2$, whence $mv^2 = 2r^2Ue$. Then

$$\nu = \frac{4r^2Up^2N^2}{X}. \qquad (5)$$

We may therefore write the average amount of energy lost per collision by an electron in the region of the gas specified by $x$ in the form

$$\frac{e}{\nu}\left(X - \frac{dU}{dx}\right), \quad \text{or} \quad \frac{Xe\left(X - \frac{dU}{dx}\right)}{4r^2Up^2N^2}.$$

This expression must equal $kUe$, where $k$ is the fraction of energy lost at a collision. Thus we obtain

$$\frac{dU}{dx} = X - \frac{4r^2p^2N^2kU^2}{X} \tag{6}$$

for the average net rate of gain of energy by an electron whose energy is $U$, expressed in equivalent volts.

If the anode is at a distance $d$ from the cathode, the average energy of the electrons reaching the anode is given by

$$\int_0^U \frac{dU}{X^2 - 4r^2p^2N^2kU^2} = \int_0^d \frac{dx}{X},$$

whence

$$U = \frac{V}{2rN\sqrt{k}pd} \frac{\epsilon^{4rN\sqrt{k}pd} - 1}{\epsilon^{4rN\sqrt{k}pd} + 1}, \tag{7}$$

where $V$ has been written for $Xd$, the total difference of potential between the anode and the cathode.

In order to adapt this relation to our experimental results in Table I., we note that we were able to ascertain the value of $U$ as soon as it became equal to the minimum ionizing potential $V_0$, whence we shall consider equation (7) when $U$ has the value $V_0$. Now $V - V_0 = s$ is the "shift" whose experimental values are given as a function of $pd$ in Table I. and Fig. 5. Solving equation (7) for this quantity, we find

$$s = V_0\left[\frac{2rN\sqrt{k}pd(\epsilon^{4rN\sqrt{k}pd} + 1)}{\epsilon^{4rN\sqrt{k}pd} - 1} - 1\right]. \tag{8}$$

In order to understand the application of this equation to the experimental results, attention should be called to the fact that the equation applies to *mean* values of the kinetic energy of the electrons, while in our experiments we detect ionization and thus determine the values of $s$ when an appreciable number of the *fastest* electrons attain the minimum ionizing energy. The following considerations enable us to take account of the difference between these points of view.

When the product $pd$ is small, so that relatively few collisions are made by each electron, there is small probability that the speed of any electron will differ appreciably from the mean speed. Consequently equation (8) may be safely used for small values of $pd$. When $pd$ is increased, however, the relation between $s$ and $pd$ approaches a linear form, which indicates that energy is being lost by collisions at almost the same rate that it is acquired from the field. This state of equilibrium is most easily expressed by placing $(dU/dx) = 0$ in equation (6), whence

$$U = \frac{X}{2rpN\sqrt{k}} \tag{9}$$

gives the mean energy of electrons in a steady state of drift in the field $X$. Strictly speaking, this state would not be reached until the electrons had moved an infinite distance through the gas, but it was reached within the limits of experimental accuracy in a number of our measurements with large values of $pd$. In other words, we were able to increase $pd$ indefinitely, keeping $X$ constant, without appreciably increasing the mean energy $U$.

Under these conditions, Langevin[1] and Boltzmann[2] have shown that the velocities of the electrons are distributed according to Maxwell's law about the mean velocity of drift. That this really applies to the case under discussion may be shown by an argument based on two equations derived by Pidduck[3] in a paper on "The Abnormal Kinetic Energy of an Electron in a Gas."

He considers electrons of mass $m$ and charge $e$ moving with a steady mean rate of drift $u_0$ in a field $X$ through a gas consisting of perfectly elastics pherical molecules of mass $M$, each set having velocities distributed according to Maxwell's law. His equations, with certain symbols changed to avoid ambiguity with the present paper, are

$$u_0 = \frac{3eX}{16N'\sigma^2}\left(\frac{6}{\pi m\lambda MV^2}\right)^{1/2},$$

$$\frac{u_0^2}{V^2} = \lambda - 1,$$

where $N'$ is the number of molecules per unit volume, $\sigma$ is the molecular radius, $V$ is the square root of the mean square velocity of molecular agitation and $\lambda$ is the ratio of the mean kinetic energy of an electron to that of a gas molecule.

[1] Ann. Chim. Phys., 105, 5, p. 245, 1905.
[2] Boltzmann, Gastheorie, Vol. 1, p. 114.
[3] Roy. Soc. Proc., 88, p. 296, 1913.

If we substitute $\pi N'\sigma^2 = pN$ in the first equation and use the second equation to eliminate $u_0$, we obtain

$$\lambda \cdot \tfrac{1}{2} M V^2 = \frac{3eX}{32pN}\sqrt{\frac{6\pi M}{m}},$$

neglecting the factor $(\lambda - 1)/\lambda$ which does not differ appreciably from unity. The first member is, by definition of $\lambda$, equal to the mean kinetic energy of an electron, which we have expressed in the form $eU$. Thus

$$U = \frac{3X}{32pN}\sqrt{\frac{6\pi M}{m}}. \tag{10}$$

By equation (2) we may replace $\sqrt{M/m}$ by $\sqrt{2/k}$. Equation (10) thus becomes

$$U = \frac{3\sqrt{12\pi}}{32}\frac{X}{pN\sqrt{k}} = 1.151\,\frac{X}{2pN\sqrt{k}}.$$

This is seen to be identical with equation (9) of the present paper, since the numerical term 1.151 is identical with $1/r$, which is the ratio of the square root of the mean square speed to the average speed in a Maxwellian distribution.

The point of this discussion is that we may apply equation (8) directly to our experimental results only when dealing with such small values of $pd$ that the maximum speed of the electrons at any point of the gas does not differ appreciably from the mean speed. Under these conditions the ratio $r$ equals unity. As the value of $pd$ increases, the ratio $r$ diminishes, approaching the value $1.151^{-1}$ as a limit. For very large values of $pd$, equation (8) becomes

$$s = V_0(2rN\sqrt{k}pd - 1). \tag{11}$$

If we know, from the characteristics of our apparatus, the least proportion of the electrons whose ionization can be detected, we may apply equation (11) to our experimental results if we give to the average energy, not the value $V_0$, but such a smaller value as will give, according to Maxwell's distribution, the necessary proportion of electrons with energies equal to or greater than the minimum ionizing energy $V_0$. In this case the constant $r$ in equation (11) should be given the value $1.151^{-1}$.

We have, therefore, two methods of using the experimental results to determine the value of $k$. Of these methods, the one utilizing very small values of $pd$ is the more direct and accurate.

*Calculation of Elasticity of Impact in Helium.*—The experimental determinations of the relation between $s$ and $pd$ are shown plotted along

Curve 1, Fig. 5. For very small values of $pd$ the points are plotted on a larger scale in Fig. 6, which includes the region marked off by the small rectangle near the origin in Fig. 5.

In equation (8) $V_0$ has the value 20 volts, and $N$ will be taken to be 8.5. This value is calculated from values of the mean free path of helium atoms at 1 mm. pressure by taking the mean free path of an electron to be $4\sqrt{2}$ times that of an atom, in accordance with Maxwell's conclusions regarding a small particle moving with relatively high speed among larger particles. Different methods of estimating the mean free path of a helium atom give somewhat different results, so that a weighted mean value of these results was used to determine the value $N = 8.5$. As a matter of fact, $N$ enters into the equation in such a way that the conclusions arrived at would not be appreciably affected if any of the individual values rather than their mean had been used. With these values equations (8) and (11) become

$$s = 20\left[\frac{17r\sqrt{kpd}(\epsilon^{34r\sqrt{kpd}} + 1)}{(\epsilon^{34r\sqrt{kpd}} - 1)} - 1\right] \tag{12}$$

and

$$s = 20(17r\sqrt{kpd} - 1). \tag{13}$$

The ratio $r = 1$ when $pd = 0$ in equation (12) and decreases gradually as $pd$ increases, approaching the value $r = 1.151^{-1}$ for equation (13).

In Fig. 5, Curve 2 represents equation (12) on the assumption that impacts are perfectly elastic, so that $k = 0.0002685$ by equation (4). It is seen to coincide with the experimental Curve 1 when $pd$ is very

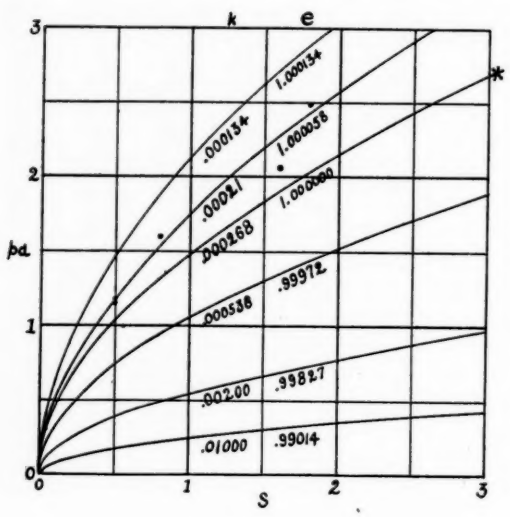


Fig. 6.

small, and at large values of $pd$ to approach the straight dotted Curve 3, which represents equation (3).

(*a*) *Use of Small Values of pd to Determine k.*—In Fig. 6 are shown graphs of equation (12) for various arbitrarily chosen values of $k$. The correct value of $k$ is the smallest value for which the curve of equation (12) lies entirely below the experimental results, approaching coincidence with the experimental curve at the origin. The reason for this choice is obvious from the discussion in the preceding section. For instance, $k$ is less than 0.002, since a curve with a smaller value of $k$ can obviously be drawn without passing above and intersecting the experimental curve. Similarly $k$ is greater than 0.000134, since this curve, near the origin, lies above the experimental curve. An examination of the relation of the curves of Fig. 6 to the plotted experimental values shows that $k$ cannot be smaller than about 0.00024 and cannot be larger than about 0.00035. Mechanical considerations show that $k$ cannot be less than 0.0002685, which represents perfect elasticity. Thus the value of $k$ is fixed with considerable certainty between 0.0002685 and 0.00035. These values of $k$, by equation (1), show that the coefficient of restitution cannot differ from unity by more than 0.01 per cent.

This degree of accuracy in the determination of $e$ seems, at first sight, impossible. It is possible because of the very small proportion of energy lost per impact, whence a very slight decrease in the degree of elasticity would greatly increase the proportion $k$ of energy lost.

(*b*) *Use of Large Values of pd to Determine k.*—A consideration of the sensitiveness and constancy characteristics of our apparatus leads us to the conclusion that a consistent increase of 5 per cent. in the electronic current is about the least increase which we could detect and take as definitely indicating a "break" in the experimental curves of Figs. 3 and 4. We will therefore take 5 per cent. to be approximately the proportion of the electrons present which must have energies equal to or greater than 20 volts in order that ionization may be detected. In a Maxwellian distribution it is found that 5 per cent. of the particles have kinetic energies equal to or greater than 2.6 times the mean energy. Thus, in the present case, 20 volts represents 2.6 times the mean energy $U$, whence the mean energy at the "break points" must have been close to $U = 7.7$ volts.

The slope of the theoretical $pd - s$ curve for large values of $pd$ is shown by equation (11) to be $(2V_0 r N \sqrt{k})^{-1}$, if the average energy were represented by $V_0$, or 20 volts for helium. We have just seen, however, that the average energy appropriate to our experiments must be taken to be about 7.7 volts. Substituting this value in place of $V_0$ and taking

$r = 1.151^{-1}$ and $N = 8.5$, we should obtain the slope of the experimental Curve 1, Fig. 5, which is very near 0.5. Thus

$$\frac{1.151}{2(7.7)8.5\sqrt{k}} = 0.5,$$

whence

$$k = 0.00031.$$

Obviously there is much greater uncertainty with regard to calculations by this method than with regard to results determined by the method previously discussed. However the order of magnitude cannot be in error, whence this method affords a confirmatory check of the results of the first method.

*An Attempt to Apply the Method to Hydrogen and Oxygen.*—Extensive series of measurements similar to those made with helium were made with carefully purified hydrogen and oxygen in the apparatus. In dealing with either of these gases it was found very difficult to determine definitely the point at which ionization begins, the "break points" in the experimental curves being much less sharply defined than in the case of helium. This was particularly true when working at small values of pressure times distance *pd*. We never found any indication of a second "break" in a curve. Furthermore, the upward inflections in the $pd - s$ curves, shown in Fig. 7, cannot be explained on the assumptions underlying equation (8). For this reason it is not deemed important to present here the original data or curves, although certain conclusions of a qualitative nature may be drawn from the results.

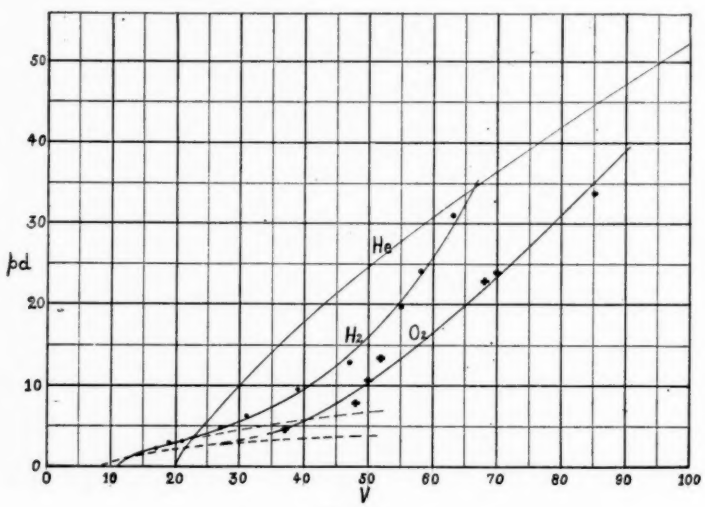


Fig. 7.

In Fig. 7 are shown the potentials which must be applied to produce ionization for various values of *pd* for oxygen and hydrogen. Curve 1 of Fig. 5 for helium is also reproduced for comparison. The feet of the curves correspond to the minimum ionizing potentials 8.5, 11.0 and 20.0 volts respectively.

If ionization is due only to the impacts of electrons, and if the average energy lost by an electron at a collision may be represented by a constant fraction of its energy for all values of energy below that necessary for ionization, then there is no reason for an upward inflection like that in the oxygen and hydrogen curves. This inflection probably indicates either ionization by positive ions or by radiation from the molecules excited by the impacts, both of which phenomena would be expected to be more effective at the larger values of *pd*. If these, or other superimposed effects, account for the upward inflection, it appears that the course of the curves, had these effects been absent, would have been somewhat as shown by the dotted lines. At any rate, the trend of the curves for the smaller values of *pd* indicates that less energy is lost at impacts in hydrogen than in oxygen, but that both of these gases are much less elastic than helium.

The difficulty in obtaining sharp "break points" in the curves for small values of *pd* and the failure to find a *series* of "break points" indicates that the group of electrons emitted from the cathode loses its homogeneity more quickly as it travels through oxygen or hydrogen than if moving through helium. This again implies that energy is lost in relatively large amounts at individual collisions, and possibly that the amount lost may depend on the angle at which the molecule is struck.

## Summary.

1. A method is developed for measuring the average fraction of its energy lost by an electron at a collision with a gas molecule for impact velocities less than the minimum ionizing velocity. This method can only be applied to a study of those gases in which the amounts of energy lost are relatively small and in which no appreciable amount of ionization is produced, within the range of pressures, distances and applied potentials used, by any agency except the impacts of the electrons.

2. Collisions of electrons with helium atoms appear to be perfectly elastic for velocities less than the velocity corresponding to 20 volts. If any energy is lost by an electron in addition to that transferred to kinetic energy of translation of the atom, such a loss is certainly less than 0.02 per cent. of the energy before impact. From this it seems safe to conclude that:

(*a*) The constituents of a helium atom are held so firmly together that they are not appreciably displaced, relatively to each other, when the atom is struck by an electron whose velocity is less than the ionizing velocity. Or, if such displacement does occur, the natural frequency of the displaced parts must be so high that there is no appreciable lag between their motion and that of the approaching and receding electron.

(*b*) There is no "minimum radiating potential" below the ionizing potential 20 volts. The only appreciable effect of the passage of the electrons through the gas is to slightly increase the mean kinetic energy of the atoms and thus slightly increase the ordinary heat radiation. The same effect on the radiation from the gas could be produced by warming it.

3. Collisions of electrons with molecules of hydrogen and oxygen are much less elastic than in the case of helium and the loss of energy is of a more complicated type, to which the method of this paper cannot be applied.

It should be of great interest to apply this method to a study of the other inert gases and to mercury vapor.

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# THE USE OF MERCURY DROPLETS IN MILLIKAN'S EXPERIMENT.

BY JOHN B. DERIEUX.

PROFESSOR MILLIKAN, in his preliminary work on *e*, tried, among droplets of other liquids, a few of mercury, and obtained, as he felt, consistent results within the limits of experimental error.[1] Other observers who have since used mercury have had difficulty with it, often getting very erratic results. Ehrenhaft obtained no consistency whatsoever.[2] Silvey, however, used it with very good success and while his results show a slight variation, he attributes it to experimental conditions, viz.: (1) evaporation of the droplets; and (2) "distortion of the electrical field by the piling up of the fallen droplets."[3]

That the first is a probable source of error may be seen by Tables I. and II. which observations I made upon two droplets in the preliminary part of this work. These tables show the extent of the evaporation often encountered. In each case the time of observation was about 30 minutes.

TABLES SHOWING THE EXCESSIVE EVAPORATION OF MERCURY DROPLETS.

TABLE I.

| Time Under Gravity. | Time Under Field. |
|---|---|
| 16.5 | 13.4 |
| 17.2 | 12.1 |
| 17.8 | 11.6 |
| | 11.6 |
| 21.4 | 35.8 |
| | 30.4 |
| | 24.4 |
| | 20.6 |
| 26.2 | 18.8 |
| 27.8 | 13.8 |
| 31.4 | 11.2 |
| | 10.6 |
| | 10.4 |
| 35.6 | 10.0 |
| | 9.8 |
| | 9.4 |
| | 9.4 |

TABLE II.

| Time Under Gravity. | Time Under Field. |
|---|---|
| 17.8 | 18.0 |
| 18.2 | 17.6 |
| 17.8 | 17.2 |
| 18.4 | 17.4 |
| 18.6 | 16.8 |
| 18.8 | 16.2 |
| 22.8 | 16.2 |
| 23.0 | 16.2 |
| 23.4 | |
| 26.2 | |
| 27.2 | 12.2 |

[1] R. A. Millikan, PHYS. REV., pp. 389, 1911.
[2] Ehrenhaft, Ann. der Phys., 44, 1914; 46, 1915.
[3] O. W. Silvey, PHYS. REV., Jan., 1916.

The second also seems a probable source, for after a series of observations the drop formed by the coalescence of the droplets often has a radius of one millimeter or more.

Liquid droplets, owing to the certainty with which their density and sphericity can be known, are preferable to solid ones and since mercury is a conductor, it is desirable to have, if possible, satisfactory results from its use in this experiment.

Accordingly this work was undertaken with the following objects in view:

1. To repeat Silvey's work to see if consistent results could be obtained with mercury if the source of error to which he attributes his variations be eliminated as far as possible.

2. If consistent results can be obtained, to determine for mercury the correction for Stokes's Law.

3. To see if the value of $e$ obtained for mercury would agree with that obtained from oil and shellac.[1]

4. To extend Silvey's work to reduced pressures and, if possible, to varying pressures on the same droplet, thus determining more accurately the slope and intercept of the line connecting $e^{2/3}$ with $1/pa$.

## APPARATUS.

The apparatus employed was of the same general form as that used by Millikan and Silvey; in fact, it was the same that the latter used, except for a few changes. Sections of it are shown in Figs. 1*A* and 1*B*.

The condenser plates $P_1$ and $P_2$ were 22 cm. in diameter and separated by glass pillars *ggg*, 1.587 cm. high. A thin ebonite strip surrounded the plates, glass windows being placed in it at *bbb*. Through the center of the upper plate were six holes about .5 mm. in diameter through which the droplets entered These holes were protected by controllable shutters $a_1$ and $a_2$. Surrounding the condenser plates was an airtight pressure cylinder $C$, 30.5 cm. in diameter, the heads of which were each secured with sixteen stud bolts. Surrounding this cylinder was a constant temperature jacket $J$, which was of heavy gas-engine oil, except during the summer, when it was found necessary to substitute water so that ice could be used to reduce the temperature. This jacket was contained in a heavy galvanized tank $T$. Leading into the upper head of the cylinder was a pipe from the iron mercury boiler $H$, which conducted the vapor from the boiler to the cylinder. In the boiler, this pipe terminated just above the mercury surface, so that the vapor that passed was the hottest and purest. Attached to the boiler was a glass

[1] J. Y. Lee, PHYS. REV., Nov., 1914.

gauge $G$ and funnel $f$ for filling. The entire attachment, boiler and gauge, was hermetically sealed into the cylinder. By means of the gauge and funnel the amount of mercury in the boiler at any time could be easily noted and when low replenished with the cylinder under reduced pressure without changing to atmospheric. This was accomplished by

Horizontal section.

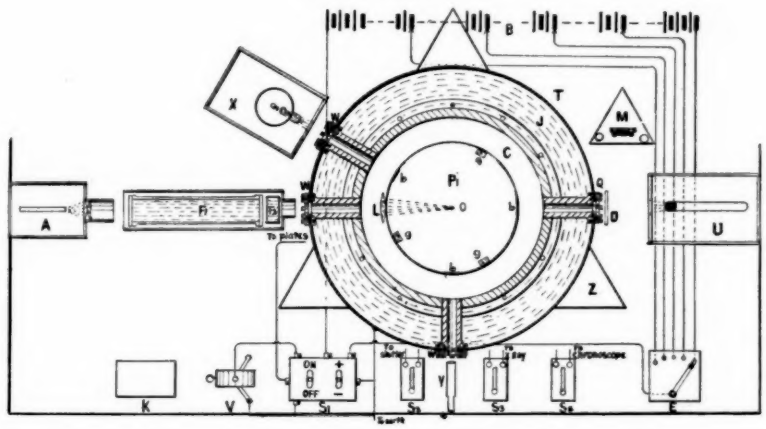


Fig. 1*A*.

Vertical section.

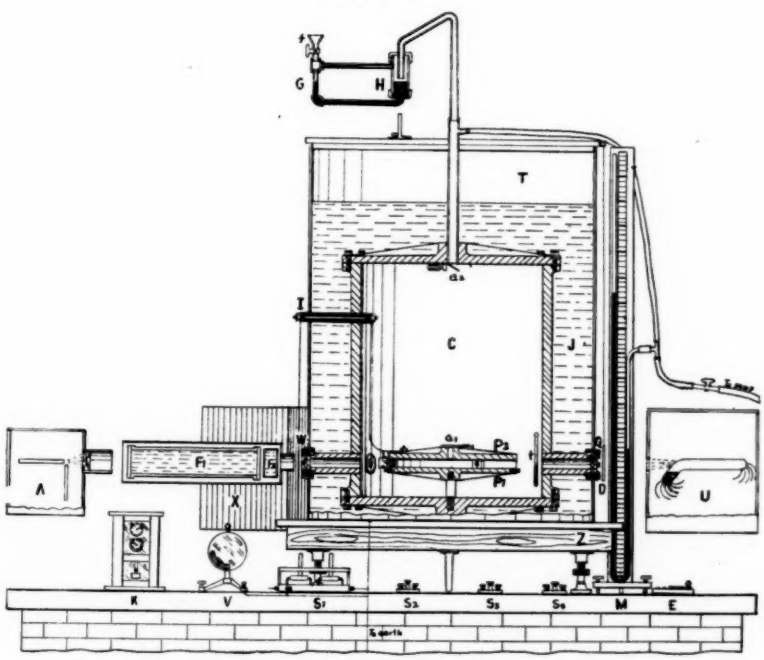


Fig. 1*B*.

first putting into the funnel the amount of mercury desired and then slightly opening the stop-cock just below, the mercury was drawn in and closing the stop-cock just before it all had passed, the admission of air was prevented. This could be done without disturbing seriously the pressure in the cylinder. The vapor from the boiler was carried into the cylinder by a blast of air admitted by the stopcock. Connected to the pipe leading into the cylinder was a line of pressure tubing to the pump and monometer $M$.

For observation, the space between the condenser plates was illuminated through a glass window $W$, by a right-angled arc $A$. The rays of the arc were focused by a cylindrical lens $L$ on the line joining the centers $O$ of the plates which was the line of fall of the droplets. The heat rays from the arc were absorbed by a water filter $F$, 80 cm. long and a cupric chloride filter $F_2$, 4 cm. thick.

A telescope $Y$ having a magnifying power of 24 and a focal length of about 25 cm., was used in observing the droplets. In the focal plane of the eyepiece was a scale, the smallest division of which corresponded to 2 mm. of fall. The extreme distance of possible fall was 1.5 cm. In focusing, the whole telescope was moved backward or forward by means of a rack and pinion, the eyepiece remaining fixed in the tube. Observations were made through a window 90 degrees from the one through which the light entered.

For timing short intervals, five to forty seconds, a Hipp chronoscope $K$, indicating to one thousandth of a second, was used. This was controlled by a switch $S_4$ at the observer's side, readings from the chronoscope being noted and recorded by an assistant. Three calibrations of this instrument were made during the work, the three giving practically the same errors which were, including personal error, + 0.07 for five seconds and − 0.10 for forty seconds with a linear relation between. The longer times were taken on a stopwatch reading to .2 of a second.

The potential of the plates was furnished by battery $B$ of 2,500 small storage cells which furnished about 2 volts each, giving a total potential of about five thousand volts. These potentials were measured by a Braune static voltmeter $V$, three calibrations of which were also in accord. The potential on the condenser was controlled by a double switch $S_1$ by means of which the condenser could be charged, grounded, and reversed. This switch was made of a large paraffin block with mercury wells for contact. A variation of the potential, when desired, was secured through a controller $E$.

For changing the charges on the droplets, an X-ray tube $X$, was used to ionize the air between the condenser plates. The rays entered through

a window about 30 degrees behind the one through which the illumination entered, or about 120 degrees from the one through which observations were made. The X-ray tube was controlled by a switch $S_3$. In the latter part of the work an ultra-violet light $U$, was used which changed the charge through photoelectric effect on the droplet, the window $Q$ through which it entered being of quartz.

The temperature of the air in the condenser was taken from a thermometer $t$ placed inside the cylinder besides the plates, readings being taken through the window $Q$. This work was done in a constant temperature room and a thermostat controlling an electrical heater kept it in the winter within .2 of a degree of constancy. In addition, there was a jacket of oil surrounding the cylinder; therefore, the temperature of the air in the condenser was kept practically constant. In the summer, when it was found necessary to substitute water for the oil so that ice could be used to keep the temperature down, the ice was applied some time before a series of observations was begun to insure uniform temperature and consequent absence of convection currents.

## EVAPORATION.

The change in the apparatus proposed by Professor Millikan to eliminate the source of error in Silvey's work was to cover the lower plate of the condenser with a pool of mercury. This should diminish the first mentioned source of error in his work, viz., evaporation, for it should keep the space surrounding the droplets saturated with mercury vapor. It should at the same time eliminate his second error, viz., the piling up of fallen droplets, by simply allowing them to become a part of the mercury of the pool.

The oil which had been used in the previous work inside the cylinder surrounding the condenser was carefully removed by several applications of benzine. To insure all trace of the benzine vapor being removed, the cylinder was left open for several hours with an electric fan playing into it.

The proposed alterations in the condenser were made and a few preliminary droplets caught to see the effect on the rate of evaporation. These showed a great reduction in it; in fact, it was almost entirely stopped. One of the droplets was held for two hours and four readings of the time required for it to fall 1 cm. under gravity taken at intervals of about thirty minutes were 48 sec., 48 sec., 46 sec., and 48 sec. It seemed as though the plan was successful. Hence regular observations were begun.

After several days, however, the rate of evaporation was found to slowly increase. Opening the cylinder to see if a cause might be surmised,

it was found that the jacketing oil had leaked in. From this it appeared that the presence of oil vapor increases the evaporation of mercury droplets and that the reduced evaporation above resulted from the absence of oil vapor rather than the presence of the mercury vapor. To verify this supposition, the tank was again thoroughly cleaned and the pool of mercury removed. Readings again taken showed about as slow evaporation as in the first instance. Observations on a droplet just preceding this change and one just following it are recorded in Tables III. and IV., respectively. In these the time under the field is omitted because at the close of the observation a broken battery connection was found and the readings, therefore, are not considered trustworthy. The entire time of observation was 20 minutes and 55 minutes, respectively, and must be considered in comparing the rates of evaporation; it is important to know too that the droplet in Table III. was not in a thoroughly saturated atmosphere of oil vapor for the reason that the leak was only slight and the oil was only present a short time.

TABLES SHOWING DECREASED EVAPORATION WITH THE ELIMINATION OF OIL VAPOR.

TABLE III.
*Presence of Oil Vapor.*

| Time Under Gravity. | Time Under Gravity. |
|---|---|
| 6.6 | 6.9 |
| 6.8 | 7.0 |
| 6.9 | 7.4 |
| 6.9 | 7.1 |
| 6.9 | 7.4 |

TABLE IV.
*Absence of Oil Vapor.*

| Time Under Gravity. | Time Under Gravity. |
|---|---|
| 9.6 | 9.7 |
| 9.7 | 9.8 |
| 9.6 | 9.7 |
| 9.6 | 9.8 |
| 9.6 | |

The increased rate of evaporation in the presence of oil vapor is in accord with Silvey's high rate of evaporation, for his work was done in an atmosphere saturated with oil vapor. This effect is probably produced by a coating formed by the condensation of the oil vapor upon the droplet. This is in accord with McKeehan's work who found that a coating of other liquids upon the surface of a mercury droplet increased its rate of evaporation.[1]

Finding that by eliminating the oil vapor the proposed alterations for reducing the evaporation were not necessary, the work was carried on without the mercury pool, the air being kept free from oil vapor.

The second source of error, the piling up of the droplets, was eliminated by frequent cleaning of the plates.

[1] L. W. McKeehan, PHYS. REV., Aug., 1916.

## MANIPULATION.

The droplets were secured by condensing the vapor from boiling mercury. The vapor was generated in boiler $H$ (Fig. 1$B$) and carried into cylinder $C$ by a blast of air. During a blast, shutter $a_2$ was held open to allow free entrance to the cylinder, shutter $a_1$ being closed to prevent the holes in the upper plate of the condenser from becoming clogged by large droplets. Following a blast, shutter $a_2$ was closed and after waiting for a few seconds for the large droplets to fall to the bottom of the cylinder, shutter $a_1$ was opened and the small droplets were seen to enter the condenser. A droplet of desired size, as judged by the velocity of fall, was selected and the potential was thrown on to the condenser. If the droplet carried a charge it responded and by manipulating the reversing section of switch $S_1$ it was drawn toward the upper plate. Shutter $a_1$ was then closed to prevent the further entrance of droplets, and the chosen one held until the field was clear and air currents had subsided.

The time required for the droplet to fall 1 cm. under gravity and that required to return under the field were observed a number of times. By means of the X-rays the charge on the droplet was changed and another series of readings was taken. These operations were repeated as often as desired. If only constant pressure results were sought, the drop was then released, another caught and similar observations taken.

In the work at varying pressures the droplet was caught at atmospheric pressure and a set of readings taken as for constant pressure. It was then drawn near the upper plate, the pump started, and the stopcock leading to it gradually opened. As the ebonite strip around the condenser fitted snugly, the main exit for the air in the condenser was through the holes in the center of the upper plate, consequently, a rising current of air was produced around the droplet. By placing it at a certain point it would be held just in balance by this current with the condenser discharged. Slightly nearer the upper plate it would be carried upward and at another point, still nearer, with the condenser reversed, it would be held in balance again, if a droplet passed beyond the last mentioned point it could be returned by closing the stopcock leading to the pump.

When the desired reduction in pressure had been made the stopcock was closed and another set of readings taken. While reductions in pressure in the work on a given droplet could be obtained as often as desired it was never repeated more than four times because evaporation, though slight, resulted in the equivalent of a different droplet if a given droplet was held too long. The values of the radii of the drop and the

values of $e_1^{2/3}$ were obtained precisely as detailed in Professor Millikan's paper[1] and the relation between $e_1^{1/3}$ and $1/pa$ graphed as there described.

## RESULTS.

In Tables V. and VI. are recorded typical readings on a droplet at atmospheric pressure, Tables VIII. and IX. at reduced pressure, and Tables XI. and XII. at varying pressures. The numbers given in the columns headed $t_g$ are the readings taken on the times of descent through 1 cm. under gravity, those headed $t_F$ give the times of ascent under the field: $1/t_F$ the reciprocals of the times of ascent, and $1/t_{F'}$ denotes the reciprocal of the time of ascent after a change in charge. Columns $n'$ contain the number of elementary units of change in charge on the droplets and are the quotients obtained by dividing the numbers in the column $(1/t_F - 1/t_{F'})$ by their greatest common divisor. Under $n$ are recorded the total number of units of charge on the droplets, determined, as in the previous case, by taking the quotients obtained by dividing the numbers under $(1/t_g - 1/t_F)$ by their greatest common divisor.

In Tables VII., X., and XVIII. are given the summaries of the results obtained at atmospheric, reduced, and varying pressures, respectively. In these Tables, under $t_g$ are recorded the average times in seconds of the fall of the droplets, under $p$, the pressure in cm. of mercury inside the pressure cylinder surrounding the condenser, under P.D. the potentials in volts between the condenser plates, and under Tem. the temperature in degrees (Cen.) of the air in the cylinder. In the columns headed $a$, are recorded the radii of the droplets, under $1/pa$, the reciprocals of the products of the radii and the pressures, and under $l/a$, the quotients of the mean free path of the molecules by the radii of the droplets. Under $n$ are recorded the extreme number of elementary units of charge upon the droplets during the observations, under $e_1^{2/3}$, the two thirds power of the values of the elementary unit of charge in electrostatic units, obtained without the application of the correction to Stokes's law, under $e^{2/3}$ the two thirds power of the values of the elementary unit after the corrections to Stokes's law, according to Millikan's method, have been applied. These last are obtained from the graph and are the intercepts on the $e_1^{2/3}$ axis of lines through the points representing the droplets and having the same slope as the general line for all of the droplets.

[1] R. A. Millikan, PHYS. REV., 1st Ser., 32, 1911; 2d Ser., II. (1913), 117; Phil. Mag., July, 1917.

## Atmospheric Pressure.

Table V.

*Drop No.* 10.

| $t_0$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_0}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_0}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| | 48.6 | | | | | | | |
| 16.96 | 49.4 | .0206 | | | | .0791 | 3 | .0264 |
| | | | .0814 | 3 | .0271 | | | |
| 17.01 | 9.91 | | | | | | | |
| 16.89 | 9.88 | .1020 | | | | .1605 | 6 | .0268 |
| 16.88 | 9.88 | | | | | | | |
| | | | .0545 | 2 | .0272 | | | |
| | 6.58 | | | | | | | |
| | 6.55 | .1565 | | | | .2150 | 8 | .0269 |
| 17.00 | 6.50 | | | | | | | |
| | 6.26 | | | | | | | |
| | | | .0279 | 1 | .0279 | | | |
| | 7.72 | | | | | | | |
| | 7.67 | | | | | | | |
| | 7.98 | .1296 | | | | .1881 | 7 | .0269 |
| | 7.99 | | | | | | | |
| | 7.63 | | | | | | | |
| | | | .1080 | 4 | .0270 | | | |
| | 46.4 | | | | | | | |
| | 47.3 | | | | | | | |
| | 47.0 | .0216 | | | | .0801 | 3 | .0267 |
| | 46.2 | | | | | | | |
| | | | .0272 | 1 | .0272 | | | |
| | 20.34 | | | | | | | |
| | 20.69 | .0488 | | | | .1073 | 4 | .0268 |
| 17.81 | 20.48 | | | | | | | |
| | | | .0267 | 1 | .0267 | | | |
| | 13.24 | | | | | | | |
| | 13.53 | .0755 | | | | .1338 | 5 | .0268 |
| 17.95 | 13.27 | | | | | | | |
| | 13.35 | | | | | | | |
| 17.40 | | | | | .0271 | | | .0269 |

Duration of exp. = 30 min.
Temp. = 23.0° C.
Pres. = 74.40 cm. Hg.
P. D. = 4,702 volts.
$a = 5.616 \times 10^{-5}$ cm.
$1/pa = 239.3$.
$e_1^{2/3} = 67.81 \times 10^{-8}$.
$e^{2/3} = 60.65 \times 10^{-8}$.

TABLE VI.

*Drop No.* 14.

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 26.19 | 15.79 | | | | | | | |
| 26.02 | 15.88 | | | | | | | |
| 26.02 | 15.88 | .0633 | | | | .1016 | 3 | .0339 |
| 26.17 | 15.81 | | | | | | | |
| 26.07 | 15.83 | | | | | | | |
| | | | .0343 | 1 | .0343 | | | |
| | 10.29 | | | | | | | |
| | 10.34 | | | | | | | |
| | 10.29 | .0976 | | | | .1359 | 4 | .0339 |
| 26.20 | 10.36 | | | | | | | |
| | 10.24 | | | | | | | |
| | | | .0685 | 2 | .0342 | | | |
| | 34.40 | | | | | | | |
| | 34.58 | | | | | | | |
| | 33.90 | .0291 | | | | .0674 | 2 | .0337 |
| 26.05 | 34.36 | | | | | | | |
| | 34.27 | | | | | | | |
| | | | .0349 | 1 | .0349 | | | |
| | 15.87 | | | | | | | |
| | 15.74 | | | | | | | |
| | 15.91 | .0630 | | | | .1013 | 3 | .0338 |
| | 15.98 | | | | | | | |
| | 16.04 | | | | | | | |
| | | | .0338 | 1 | .0338 | | | |
| | 34.21 | | | | | | | |
| | 34.27 | | | | | | | |
| | 34.16 | .0292 | | | | .0675 | 2 | .0337 |
| 26.19 | 34.18 | | | | | | | |
| | 34.29 | | | | | | | |
| | | | .0333 | 1 | .0333 | | | |
| | 15.90 | | | | | | | |
| | 16.06 | | | | | | | |
| | 16.10 | .0625 | | | | .1008 | 3 | .0336 |
| | 16.03 | | | | | | | |
| 26.11 | | | | | .0341 | | | .0337 |

Duration of exp. = 55 min.
Temp. = 23.0° C.
Pres. = 74.27 cm. Hg.
P. D. = 4,630 volts.
$a = 4.521 \times 10^{-5}$ cm.
$1/pa = 297.9$.
$e_1^{2/3} = 69.87 \times 10^{-8}$.
$e^{2/3} = 60.71 \times 10^{-8}$.

TABLE VII.

*Results at Atmospheric Pressure.*

| No. | Tem. °C. | $p$ (Cm. Hg) | P. D. (Volts) | $t_g$ (Sec.) | $a \times 10^5$ (Cm.) | $\frac{l}{p a}$ | $n$ | $e_1^{\frac{2}{3}} \times 10^5$ | $e^{\frac{2}{3}} \times 10^5$ |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 22.9 | 75.10 | 4,580 | 4.56 | 11.230 | 118.6 | 23–40 | 64.96 | 61.36 |
| 2 | 23.0 | 75.10 | 4,475 | 6.02 | 9.764 | 136.4 | 13–48 | 65.30 | 61.16 |
| 3 | 23.0 | 74.92 | 4,645 | 7.53 | 8.695 | 153.5 | 10–26 | 65.76 | 61.10 |
| 4 | 23.0 | 74.61 | 4,690 | 8.09 | 8.364 | 160.2 | 9–13 | 66.21 | 61.34 |
| 5 | 23.0 | 73.81 | 3,375 | 9.66 | 7.661 | 176.9 | 8–19 | 66.01 | 60.65 |
| 6 | 23.0 | 74.61 | 4,650 | 9.77 | 7.575 | 177.4 | 6–14 | 66.75 | 61.36 |
| 7 | 23.0 | 74.88 | 4,930 | 11.84 | 6.882 | 194.1 | 4–11 | 66.79 | 60.90 |
| 8 | 23.0 | 75.33 | 3,700 | 12.19 | 6.778 | 195.8 | 10–25 | 66.99 | 61.04 |
| 9 | 23.0 | 75.64 | 4,425 | 17.21 | 5.660 | 233.6 | 4–15 | 67.87 | 60.77 |
| 10 | 23.0 | 74.40 | 4,702 | 17.40 | 5.616 | 239.3 | 3– 8 | 67.81 | 60.54 |
| 11 | 22.8 | 75.12 | 4,690 | 18.73 | 5.413 | 246.0 | 2– 6 | 68.10 | 60.63 |
| 12 | 22.9 | 75.81 | 4,845 | 21.20 | 5.046 | 261.5 | 3– 7 | 69.50 | 61.56 |
| 13 | 23.0 | 74.44 | 4,690 | 22.00 | 4.953 | 271.2 | 4– 7 | 69.28 | 61.04 |
| 14 | 23.0 | 74.27 | 4,630 | 26.11 | 4.521 | 297.9 | 2– 4 | 69.87 | 60.83 |
| 15 | 23.1 | 74.27 | 4,500 | 34.60 | 3.852 | 349.5 | 2– 4 | 71.78 | 61.16 |
| 16 | 23.0 | 74.45 | 3,935 | 39.60 | 3.604 | 372.6 | 1– 3 | 72.74 | 61.42 |
| 17 | 23.0 | 75.98 | 3,905 | 40.58 | 3.563 | 369.5 | 1– 4 | 72.62 | 61.39 |
| 18 | 22.9 | 74.83 | 3,775 | 48.04 | 3.244 | 411.8 | 1– 2 | 74.05 | 61.54 |
| 19 | 23.0 | 74.33 | 4,105 | 49.80 | 3.195 | 421.0 | 1– 2 | 73.47 | 60.68 |
| Mean | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 61.08 |

## REDUCED PRESSURES.

TABLE VIII.

*Drop No.* 33.

| $t_0$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_0}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_0}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 27.38 | 24.23 | | | | | | | |
| 27.43 | 24.41 | .0410 | | | | .0773 | 2 | .0386 |
| | | | .0391 | 1 | .0391 | | | |
| | 12.54 | | | | | | | |
| 27.64 | 12.47 | .0801 | | | | .1164 | 3 | .0388 |
| | 12.47 | | | | | | | |
| | | | .0383 | 1 | .0380 | | | |
| | 23.74 | .0418 | | | | | | |
| 28.15 | 23.46 | .0426 | | | | .0778 | 2 | .0389 |
| | | | .0394 | 1 | .0394 | | | |
| | 310. | .0032 | | | | | | |
| | 300. | .0033 | | | | .0385 | 1 | .0385 |
| | | | .0408 | 1 | .0408 | | | |
| | 22.61 | .0441 | | | | | | |
| 28.80 | 22.21 | | | | | .0793 | 2 | .0396 |
| | 22.06 | .0453 | | | | | | |
| | | | .0403 | 1 | .0403 | | | |
| | 11.70 | | | | | | | |
| 29.28 | 11.73 | .0856 | | | | .1196 | 3 | .0399 |
| | 11.61 | | | | | | | |
| | | | .0395 | 1 | .0395 | | | |
| | 21.59 | .0461 | | | | | | |
| 29.65 | 21.28 | | | | | .0807 | 2 | .0403 |
| | 20.92 | .0478 | | | | | | |
| | | | .0406 | 1 | .0406 | | | |
| | 138. | | | | | | | |
| | 138. | .0072 | | | | .0405 | 1 | .0405 |
| | | | .0406 | 1 | .0406 | | | |
| | 20.8 | .0478 | | | | | | |
| 30.3 | 20.5 | | | | | .0820 | 2 | .0410 |
| | 20.0 | .0502 | | | | | | |
| | | | .0405 | 1 | .0405 | | | |
| | 10.92 | .0907 | | | | | | |
| 31.12 | 10.76 | | | | | .1229 | 3 | .0410 |
| | 10.79 | .0927 | | | | | | |
| | | | .0412 | 1 | .0412 | | | |
| | 19.38 | | | | | | | |
| 30.95 | 19.55 | | | | | | | |
| | 19.21 | .0515 | | | | .0837 | 2 | .0418 |
| | | | .0414 | 1 | .0412 | | | |
| 31.68 | 99.0 | .0101 | | | | .0418 | 1 | .0418 |
| | 98.0 | | | | | | | |
| 29.50 | | | | | .0401 | | | .0401 |

Duration of exp. = 35 min.
Temp. = 23.1° C.
Pres. = 56.11 cm. Hg.
P. D. = 4,590 volts.

$a = 4.086 \times 10^{-5}$ cm.
$1/pa = 436.2$.
$e_1^{2/3} = 75.37 \times 10^{-8}$.
$e^{2/3} = 62.00 \times 10^{-8}$.

TABLE IX.

*Drop No.* 35.

| $t_0$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_0}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_0}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| | 9.72 | | | | | | | |
| | 9.74 | .0141 | | | | .1661 | 6 | .0277 |
| 16.16 | 9.69 | | | | | | | |
| | | | .0554 | 2 | .0277 | | | |
| | 20.50 | | | | | | | |
| 16.20 | 20.55 | .0487 | | | | .1101 | 4 | .0275 |
| | | | .0546 | 2 | .0273 | | | |
| | − 168. (fall) | .0059 | | | | .0454 | 2 | .0272 |
| | | | .0545 | 2 | .0272 | | | |
| | 20.55 | .0486 | | | | | | |
| 16.40 | 20.43 | | | | | .1103 | 4 | .0276 |
| | 20.17 | .0498 | | | | | | |
| | | | .0274 | 1 | .0274 | | | |
| | 13.09 | | | | | .1370 | 5 | .0274 |
| 16.45 | 12.87 | .0772 | | | | | | |
| | 12.96 | | | | | | | |
| 16.30 | | | .0274 | | | | | .0275 |

Duration of exp. = 15 min.
Temp. = 23.3° C.
Pres. = 56.86 cm. Hg.
P. D. = 4,645 volts.
$a = 5.673 \times 10^{-5}$ cm.
$1/pa = 310.0$.
$e_1^{2/3} = 71.00 \times 10^{-6}$.
$e^{2/3} = 61.50 \times 10^{-6}$.

TABLE X.

*Results at Reduced Pressures.*

| No. | Tem. ° C. | $p$ (Cm. Hg) | P. D. (Volts) | $t_p$ (Sec.) | $a \times 10^5$ (Cm.) | $\frac{1}{pa}$ | $n$ | $e_1^{\frac{2}{3}} \times 10^5$ | $e^{\frac{2}{3}} \times 10^5$ |
|---|---|---|---|---|---|---|---|---|---|
| 20 | 22.9 | 33.85 | 4,560 | 16.85 | 5.295 | 558.0 | 2– 5 | 79.17 | 61.97 |
| 21 | 22.9 | 33.65 | 4,775 | 31.75 | 3.597 | 826.2 | 1– 2 | 91.12 | (65.64) |
| 22 | 23.0 | 30.89 | 4,545 | 46.25 | 2.817 | 1150.0 | 2– 4 | 102.00 | (66.54) |
| 23 | 22.7 | 40.49 | 4,690 | 22.45 | 4.648 | 531.4 | 2– 5 | 77.23 | 60.85 |
| 24 | 23.0 | 57.70 | 4,560 | 32.58 | 3.885 | 446.2 | 1– 4 | 76.25 | 62.49 |
| 25 | 23.0 | 51.44 | 4,520 | 10.10 | 7.396 | 260.7 | 5– 9 | 68.91 | 60.87 |
| 26 | 23.1 | 52.07 | 4,480 | 27.75 | 4.286 | 445.3 | 1– 4 | 73.30 | 59.57 |
| 27 | 23.0 | 56.76 | 4,645 | 21.62 | 4.876 | 361.3 | 2– 6 | 72.64 | 61.50 |
| 28 | 23.0 | 55.44 | 4,610 | 26.75 | 4.424 | 407.8 | 1– 4 | 71.32 | 58.75 |
| 29 | 23.5 | 51.73 | 4,195 | 11.66 | 6.875 | 281.2 | 5–12 | 67.86 | 59.19 |
| 30 | 23.0 | 56.24 | 4,625 | 16.03 | 5.685 | 312.8 | 3– 8 | 72.28 | 62.63 |
| 31 | 23.4 | 60.96 | 4,610 | 14.00 | 6.173 | 265.3 | 3– 7 | 69.39 | 61.21 |
| 32 | 23.1 | 50.43 | 4,600 | 37.50 | 3.556 | 557.7 | 1– 3 | 78.74 | 61.54 |
| 33 | 23.1 | 56.11 | 4,590 | 29.50 | 4.086 | 436.2 | 1– 3 | 75.37 | 61.92 |
| 34 | 23.0 | 53.52 | 4,630 | 36.10 | 3.722 | 502.1 | 1– 3 | 76.02 | 60.54 |
| 35 | 23.2 | 56.86 | 4,645 | 16.30 | 5.673 | 310.0 | 2– 6 | 71.00 | 61.44 |
| 36 | 23.3 | 58.00 | 4,600 | 23.90 | 4.588 | 375.8 | 1– 5 | 74.30 | 62.72 |
| 37 | 23.7 | 58.11 | 4,570 | 50.20 | 2.994 | 574.8 | 1– 2 | 80.13 | 62.41 |
| 38 | 23.4 | 46.49 | 4,570 | 28.30 | 4.129 | 521.0 | 1– 2 | 76.81 | 60.75 |
| 39 | 24.3 | 32.04 | 4,580 | 8.96 | 7.570 | 412.3 | 5–12 | 73.31 | 60.59 |
| 40 | 24.7 | 33.36 | 4,555 | 14.30 | 5.782 | 518.4 | 3– 7 | 78.34 | 62.36 |
| 41 | 25.3 | 74.71 | 4,600 | 24.01 | 4.671 | 286.5 | 2– 6 | 71.36 | 62.53 |
| 42 | 26.2 | 74.67 | 4,565 | 14.80 | 6.113 | 219.1 | 3– 5 | 68.00 | 61.25 |
| 43 | 26.5 | 74.67 | 4,520 | 35.65 | 3.767 | 355.7 | 1– 3 | 71.85 | 60.89 |
| 44 | 26.5 | 38.57 | 4,570 | 25.50 | 4.227 | 613.4 | 1– 3 | 82.12 | (63.21) |
| 45 | 25.7 | 41.69 | 4,590 | 14.94 | 5.806 | 413.1 | 3– 7 | 74.44 | 61.70 |
| 46 | 27.6 | 45.37 | 4,515 | 36.33 | 3.612 | 610.3 | 1– 3 | 79.52 | 60.71 |
| 47 | 28.6 | 59.23 | 4,515 | 10.08 | 7.404 | 227.9 | 5–10 | 67.92 | 60.89 |
| 48 | 28.5 | 74.89 | 4,560 | 15.65 | 5.965 | 223.9 | 3– 7 | 67.20 | 60.30 |
| 49 | 28.0 | 74.89 | 4,635 | 38.16 | 3.717 | 358.4 | 1– 3 | 71.53 | 60.38 |
| 50 | 22.9 | 74.93 | 4,560 | 15.73 | 5.923 | 225.3 | 3– 5 | 67.74 | 60.79 |
| Mean | ..... | ..... | ..... | ..... | ..... | ..... | ..... | ..... | 61.16 |

Values of $e^{2/3}$ inclosed in parenthesis are considered as beyond the breaking point of the curve and are not used either in the graph or the mean.

## VARYING PRESSURES.

TABLE XI.

*Drop No. 59A.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 23.27 | 52.6 | .0190 | | | | .0617 | 2 | .0308 |
| 23.30 | | | .0313 | 1 | .0313 | | | |
| 23.34 | 19.66 | | | | | | | |
| | 19.97 | .0503 | | | | .0930 | 3 | .0310 |
| | 20.04 | | | | | | | |
| | | | .0313 | 1 | .0313 | | | |
| 23.29 | 12.35 | | | | | | | |
| | 12.44 | | | | | | | |
| | 12.25 | .0816 | | | | .1243 | 4 | .0311 |
| | 12.33 | | | | | | | |
| | | | .0623 | 2 | .0311 | | | |
| | 51.6 | | | | | | | |
| 23.40 | 51.8 | .0193 | | | | .0620 | 2 | .0310 |
| | | | .0317 | 1 | .0316 | | | |
| | 20.02 | | | | | | | |
| | 19.75 | .0511 | | | | .0936 | 3 | .0312 |
| | 19.78 | | | | | | | |
| | | | .0314 | 1 | .0315 | | | |
| 23.60 | 12.22 | | | | | | | |
| 23.58 | 12.28 | | | | | | | |
| | 12.25 | .0824 | | | | .1251 | 4 | .0313 |
| 23.44 | 12.11 | | | | | | | |
| 23.36 | | | | | .0314 | | | .0311 |

Duration of exp. = 30 min.
Temp. = 23.0° C.
Pres. = 75.09 cm. Hg.
P. D. = 4,625 volts.
$a = 4.815 \times 10^{-5}$ cm.
$1/pa = 276.0$.
$e_1^{2/3} = 69.00 \times 10^{-8}$.
$e^{2/3} = 60.6 \times 10^{-8}$.

TABLE XII.

*Drop No. 59B.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 20.53 | 39.67 | | | | | | | |
| 20.76 | 39.78 | .0251 | | | | .0737 | 2 | .0368 |
| | | | .0370 | 1 | .0370 | | | |
| | 16.22 | | | | | | | |
| | 16.14 | .0621 | | | | .1107 | 3 | .0369 |
| | 16.09 | | | | | | | |
| | | | .0379 | 1 | .0379 | | | |
| 20.79 | 10.15 | | | | | | | |
| | 10.09 | | | | | | | |
| | 10.03 | .1000 | | | | .1482 | 4 | .0374 |
| | 10.07 | | | | | | | |
| | | | .0744 | 2 | .0372 | | | |
| 20.90 | 38.97 | | | | | | | |
| | 38.80 | .0256 | | | | .0736 | 2 | .0368 |
| | 38.96 | | | | | | | |
| | | | .0369 | 1 | .0369 | | | |
| 20.82 | 16.15 | | | | | | | |
| | 16.13 | .0625 | | | | .1104 | 3 | .0368 |
| | 16.05 | | | | | | | |
| | | | .0375 | 1 | .0375 | | | |
| 20.84 | 10.10 | | | | | | | |
| 20.73 | 9.97 | .1000 | | | | .1480 | 4 | .0373 |
| | 10.15 | | | | | | | |
| | 9.98 | | | | | | | |
| 20.80 | | | | | .0373 | | | .0370 |

Duration of exp. = 35 min.
Temp. = 23.0° C.
Pres. = 34.81 cm. Hg.
P. D. = 4,565 volts.
$a$ = 4.704 × $10^{-5}$ cm.
$1/pa$ = 609.6.
$e_1^{2/3}$ = 81.14 × $10^{-8}$.
$e^{2/3}$ = 62.6 × $10^{-8}$.

TABLE XIII.

*Drop No. 59C.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 16.96 | 30.84 | | | | | | | |
| 16.89 | 31.10 | .0322 | | | | .0897 | 2 | .0449 |
| | 31.03 | | | | | | | |
| | | | .0466 | 1 | .0466 | | | |
| 17.31 | 12.78 | | | | | | | |
| | 12.69 | | | | | | | |
| | 12.93 | .0788 | | | | .1363 | 3 | .0454 |
| | 12.65 | | | | | | | |
| | | | .0469 | 1 | .0469 | | | |
| 17.19 | 7.98 | | | | | | | |
| | 8.15 | | | | | | | |
| | 7.95 | .1257 | | | | .1832 | 4 | .0458 |
| | 8.07 | | | | | | | |
| | 7.98 | | | | | | | |
| | | | .0921 | 2 | .0460 | | | |
| 17.40 | 29.58 | | | | | | | |
| | 30.15 | .0336 | | | | .0911 | 2 | .0455 |
| | 29.48 | | | | | | | |
| | | | .0460 | 1 | .0460 | | | |
| 17.41 | 12.63 | | | | | | | |
| | 12.88 | .0796 | | | | .1371 | 3 | .0457 |
| | 12.54 | | | | | | | |
| | 12.70 | | | | | | | |
| | | | .0463 | 1 | .0460 | | | |
| 17.52 | 8.00 | | | | | | | |
| 17.39 | 8.00 | | | | | | | |
| 17.37 | 8.13 | .1256 | | | | .1831 | 4 | .0458 |
| | 8.05 | | | | | | | |
| | 8.13 | | | | | | | |
| 17.39 | | | | | .0465 | | | .0455 |

Duration of exp. = 25 min.
Temp. = 23.0° C.
Pres. = 19.63 cm. Hg.
P. D. = 4,540 volts.
$a = 4.641 \times 10^{-5}$ cm.
$1/pa = 1097.0$.
$e_1^{2/3} = 99.86 \times 10^{-8}$.
$e^{2/3} = 65.8 \times 10^{-8}$.

TABLE XIV.

*Drop No. 60A.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 19.96 | 37.23 | | | | | | | |
| 19.77 | 36.79 | .0269 | | | | .0776 | 3 | .0259 |
| | 37.37 | | | | | | | |
| | | | .0269 | 1 | .0269 | | | |
| 19.96 | 18.59 | | | | | | | |
| | 18.77 | .0538 | | | | .1045 | 4 | .0261 |
| | 18.48 | | | | | | | |
| | | | .0266 | 1 | .0266 | | | |
| 19.76 | 12.54 | | | | | | | |
| | 12.51 | .0804 | | | | .1311 | 5 | .0262 |
| | 12.47 | | | | | | | |
| | 12.51 | | | | | | | |
| | | | .0541 | 2 | .0270 | | | |
| 19.73 | 37.37 | | | | | | | |
| | 37.90 | .0263 | | | | .0770 | 3 | .0257 |
| | 38.41 | | | | | | | |
| | 38.07 | | | | | | | |
| | | | .0273 | 1 | .0273 | | | |
| 19.90 | 18.61 | | | | | | | |
| | 18.80 | .0536 | | | | .1043 | 4 | .0261 |
| | 18.57 | | | | | | | |
| | | | .0267 | 1 | .0267 | | | |
| 19.69 | 12.47 | | | | | | | |
| 19.64 | 12.52 | | | | | | | |
| | 12.59 | .0803 | | | | .1310 | 5 | .0262 |
| 19.87 | 12.62 | | | | | | | |
| 19.80 | 12.50 | | | | | | | |
| 19.76 | | | | | .0269 | | | .0260 |

Duration of exp. = 30 min.
Temp. = 22.9° C.
Pres. = 75.10 cm. Hg.
P. D. = 4,612 volts.
$a = 5.445 \times 10^{-5}$ cm.
$1/pa = 247.7$.
$e_1^{2/3} = 65.80 \times 10^{-8}$.
$e^{2/3} = 58.3 \times 10^{-8}$.

TABLE XV.

*Drop No. 60B.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 18.02 | 33.76 | | | | | | | |
| 18.13 | 33.81 | .0293 | | | | .0858 | 3 | .0286 |
| | 34.29 | | | | | | | |
| | | | .0296 | 1 | .0296 | | | |
| 17.93 | 17.11 | | | | | | | |
| | 17.08 | .0589 | | | | .1147 | 4 | .0287 |
| | 16.82 | | | | | | | |
| | | | .0591 | 2 | .0295 | | | |
| | 8.54 | | | | | | | |
| | 8.69 | | | | | | | |
| | 8.35 | | | | | | | |
| 17.91 | 8.54 | .1180 | | | | .1738 | 6 | .0289 |
| | 8.69 | | | | | | | |
| | 8.33 | | | | | | | |
| | 8.54 | | | | | | | |
| | 8.45 | | | | | | | |
| | | | .0887 | 3 | .0296 | | | |
| 17.88 | 33.84 | | | | | | | |
| | 33.82 | .0293 | | | | .0851 | 3 | .0284 |
| | 34.54 | | | | | | | |
| | | | .0284 | 1 | .0284 | | | |
| 17.95 | 17.48 | | | | | | | |
| | 17.36 | .0577 | | | | .1135 | 4 | .0284 |
| | 17.31 | | | | | | | |
| | | | .0299 | 1 | .0299 | | | |
| 18.01 | 11.51 | | | | | | | |
| | 11.62 | .0876 | | | | .1434 | 5 | .0287 |
| | 11.54 | | | | | | | |
| | 11.22 | | | | | | | |
| | | | .0304 | 1 | .0304 | | | |
| 18.06 | 8.51 | | | | | | | |
| | 8.62 | .1180 | | | | .1738 | 6 | .0289 |
| | 8.46 | | | | | | | |
| | | | .0305 | 1 | .0305 | | | |
| 18.02 | 11.53 | | | | | | | |
| 18.01 | 11.56 | .0875 | | | | .1433 | 5 | .0287 |
| | 11.45 | | | | | | | |
| 17.92 | | | | | .0297 | | | .0287 |

Duration of exp. = 45 min.
Temp. = 23.0° C.
Pres. = 43.07 cm. Hg.
P. D. = 4,600 volts.
$a = 5.377 \times 10^{-5}$ cm.
$1/pa = 431.8$.
$e_1^{2/3} = 72.23 \times 10^{-6}$.
$e^{2/3} = 59.1 \times 10^{-6}$.

TABLE XVI.

*Drop No. 60C.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| 16.55 | 31.77 | | | | | | | |
| 16.37 | 32.49 | .0312 | | | | .0926 | 3 | .0309 |
| | 31.75 | | | | | | | |
| | | | .0322 | 1 | .0322 | | | |
| | − 1000 (falling) | .0010 | | | | .0622 | 2 | .0310 |
| | | | .0646 | 2 | .0323 | | | |
| 16.35 | 15.72 | | | | | | | |
| | 15.72 | .0636 | | | | .1248 | 4 | .0312 |
| | 15.78 | | | | | | | |
| | | | .0646 | 2 | .0323 | | | |
| | − 1000 (falling) | .0010 | | | | .0622 | 2 | .0311 |
| | | | .0324 | 1 | .0324 | | | |
| 16.43 | 31.72 | | | | | | | |
| | 32.23 | .0314 | | | | .0926 | 3 | .0309 |
| | 31.46 | | | | | | | |
| | | | .0327 | 1 | .0324 | | | |
| 16.39 | 15.73 | | | | | | | |
| | 15.52 | .0641 | | | | .1253 | 4 | .0313 |
| 16.44 | 15.63 | | | | | | | |
| 16.34 | | | | | .0323 | | | .0311 |

Duration of exp. = 30 min.
Temp. = 23.1° C.
Pres. = 29.95 cm. Hg.
P. D. = 4,590 volts.
$a$ = 5.394 × $10^{-5}$ cm.
$1/pa$ = 619.0.
$e_1^{2/3}$ = 78.84 × $10^{-8}$.
$e^{2/3}$ = 60.2 × $10^{-8}$.

TABLE XVII.

*Drop No. 60D.*

| $t_g$ | $t_F$ | $\frac{1}{t_F}$ | $\frac{1}{t_F}-\frac{1}{t_{F'}}$ | $n'$ | $\frac{1}{n'}\left(\frac{1}{t_F}-\frac{1}{t_{F'}}\right)$ | $\frac{1}{t_g}+\frac{1}{t_F}$ | $n$ | $\frac{1}{n}\left(\frac{1}{t_g}+\frac{1}{t_F}\right)$ |
|---|---|---|---|---|---|---|---|---|
| | − 1000 (falling) | .0010 | | | | .0730 | 2 | .0365 |
| | | | .0390 | 1 | .0390 | | | |
| 13.79 | | | | | | | | |
| 13.79 | | | | | | | | |
| 13.93 | 26.29 | | | | | | | |
| | 26.19 | .0383 | | | | .1107 | 3 | .0369 |
| | 25.86 | | | | | | | |
| | | | .0392 | 1 | .0392 | | | |
| 13.78 | 12.91 | | | | | | | |
| | 12.78 | .0775 | | | | .1499 | 4 | .0375 |
| | 13.28 | | | | | | | |
| | 12.94 | | | | | | | |
| | | | .0780 | 2 | .0390 | | | |
| | − 1000 (falling) | .0010 | | | | .0730 | 2 | .0365 |
| | | | .0390 | 1 | .0390 | | | |
| 14.00 | 26.23 | | | | | | | |
| | 26.40 | .0381 | | | | .1105 | 3 | .0368 |
| | 26.14 | | | | | | | |
| | | | .0386 | 1 | .0386 | | | |
| 13.84 | 12.96 | | | | | | | |
| 13.88 | 13.27 | .0767 | | | | .1491 | 4 | .0373 |
| | 13.11 | | | | | | | |
| | 13.02 | | | | | | | |
| 13.80 | | | | | .0389 | | | .0369 |

Duration of exp. = 20 min.
Temp. = 23.1° C.
Pres. = 18.00 cm. Hg.
P. D. = 4,580 volts.
$a = 5.363 \times 10^{-5}$ cm.
$1/pa = 1036.0$.
$e_1^{2/3} = 94.19 \times 10^{-8}$.
$e^{2/3} = 61.8 \times 10^{-8}$.

TABLE XVIII.

*Results at Varying Pressures.*

| No. | Tem. °C. | $p$ (Cm. Hg) | P. D. (Volts) | $t_y$ (Sec.) | $a \times 10^5$ (Cm.) | $\frac{1}{p_1 a}$ | $n$ | $e_1^{\frac{2}{3}} \times 10^8$ | $\frac{l}{a} \times 10^2$ |
|---|---|---|---|---|---|---|---|---|---|
| 51 *A* | 23.0 | 74.89 | 4,560 | 10.53 | 7.328 | 182.3 | 5–9 | 66.98 | 13.04 |
| " *B* | 23.0 | 29.49 | 4,550 | 13.80 | 5.869 | 577.9 | 5–6 | 79.57 | 41.35 |
| 52 *A* | 23.0 | 74.64 | 4,440 | 12.54 | 6.604 | 199.5 | 4–8 | 65.65 | 14.24 |
| " *B* | 23.0 | 31.31 | 4,420 | 18.44 | 5.163 | 618.7 | 2–5 | 76.10 | 44.27 |
| 53 *A* | 22.9 | 74.67 | 4,550 | 12.70 | 6.829 | 196.1 | 5–9 | 63.16 | 14.03 |
| " *B* | 23.0 | 44.30 | 4,560 | 12.62 | 6.631 | 340.4 | 5–7 | 67.30 | 24.35 |
| 54 *A* | 22.9 | 74.92 | 4,580 | 22.83 | 4.939 | 270.3 | 2–5 | 67.05 | 19.34 |
| " *B* | 23.0 | 40.92 | 4,580 | 26.51 | 4.307 | 567.4 | 1–3 | 76.03 | 40.60 |
| 55 *A* | 22.9 | 74.74 | 4,610 | 19.36 | 5.381 | 248.6 | 2–5 | 66.68 | 17.79 |
| " *B* | 22.9 | 32.79 | 4,600 | 18.38 | 5.184 | 588.3 | 2–5 | 75.61 | 42.09 |
| 56 *A* | 22.8 | 75.17 | 4,555 | 14.26 | 6.151 | 216.3 | 3–5 | 69.34 | 15.47 |
| " *B* | 23.0 | 29.84 | 4,555 | 13.70 | 5.727 | 585.2 | 3–5 | 83.30 | 41.87 |
| " *C* | 23.1 | 16.46 | 4,550 | 11.33 | 5.641 | 1076.0 | 2–4 | 103.80 | 77.00 |
| 57 *A* | 23.0 | 75.11 | 4,545 | 14.61 | 6.117 | 217.6 | 3–6 | 68.52 | 75.57 |
| " *B* | 23.4 | 36.16 | 4,547 | 13.94 | 5.902 | 468.6 | 3–5 | 76.91 | 33.53 |
| 58 *A* | 23.0 | 74.69 | 4,597 | 24.90 | 4.763 | 280.6 | 1–3 | 65.98 | 20.07 |
| " *B* | 23.5 | 33.29 | 4,585 | 21.75 | 4.764 | 630.5 | 1–3 | 76.78 | 45.10 |
| 59 *A* | 23.0 | 75.09 | 4,625 | 23.36 | 4.815 | 276.0 | 2–4 | 69.00 | 19.74 |
| " *B* | 23.0 | 34.81 | 4,565 | 20.80 | 4.704 | 609.6 | 2–4 | 81.14 | 43.61 |
| " *C* | 23.0 | 19.63 | 4,540 | 17.39 | 4.641 | 1097.0 | 2–4 | 99.86 | 79.97 |
| 60 *A* | 22.9 | 75.10 | 4,612 | 19.76 | 5.445 | 247.7 | 3–5 | 65.80 | 17.72 |
| " *B* | 23.0 | 43.07 | 4,600 | 17.92 | 5.377 | 431.8 | 3–6 | 72.23 | 30.89 |
| " *C* | 23.1 | 29.95 | 4,590 | 16.34 | 5.294 | 619.0 | 2–4 | 78.84 | 44.29 |
| " *D* | 23.1 | 18.00 | 4,580 | 13.80 | 5.363 | 1036.0 | 2–4 | 94.19 | 74.11 |
| 61 *A* | 23.7 | 74.84 | 4,607 | 14.80 | 6.074 | 220.0 | 3–5 | 68.55 | 15.74 |
| " *B* | 23.9 | 41.26 | 4,590 | 13.50 | 6.085 | 398.3 | 3–5 | 74.89 | 28.49 |

TABLE XIX.

*Slopes at Varying Pressures.*

| Drop No.. | 51. | 52. | 53. | 54. | 55. | 56. | 57. | 58. | 59. | 60. | 61. | Mean. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Slope $\times 10^8$. | 44.46 | 34.80 | 40.13 | 42.25 | 36.76 | 52.88 | 46.71 | 43.15 | 50.86 | 49.08 | 49.73 | 44.63 |
| Value of $A$..... | | | | | | | | | | | | .730 |

$$\text{Slope} = \frac{e_1^{2/3\prime\prime} - e_1^{2/3\prime}}{\frac{l}{a}}, \qquad A = \frac{\text{Slope}}{61.13 \times 10^{-8}}.$$

## SUMMARY.

I. Consistent results are obtainable from the use of mercury droplets if the necessary precautions are taken. The greater variation in the results at reduced and varying pressures, are due, I think, partly at least, to the higher rate of evaporation present during this part of the work. The extent of it, in some instances, may be seen from Table VIII.

II. As the correction term constant, $A$, for Stokes's law, the results at atmospheric pressure give .695, reduced pressures .705, and varying

Graph from results at atmospheric pressure.

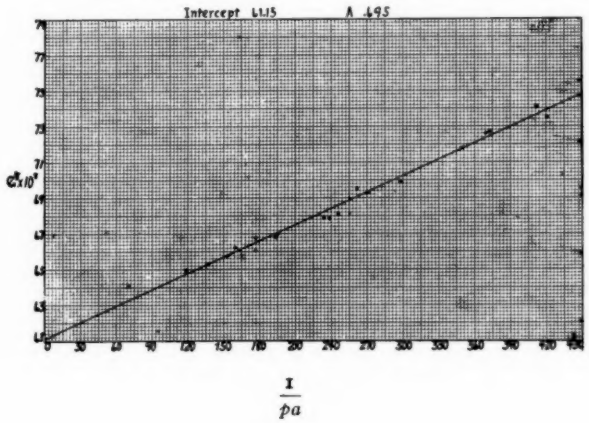


Fig. 2.

pressures .730. The greater value in the last instance is due, I believe, to a change in the surface of a droplet between the first and last observations upon it. It is probable that an oxide film forming upon it increases the coefficient of slip toward that of solid spheres. In taking the mean, therefore, I would give the above values weights of 3, 2, 1, respectively, thus giving $A$ the value .704.

III. The value of $e^{2/3}$ obtained from the mean of the results at atmospheric and reduced pressures is $61.12 \times 10^{-8}$ which is practically the same as that obtained from the use of oil, *i. e.*, $61.13 \times 10^{-8}$.

Graph from results at reduced pressures.

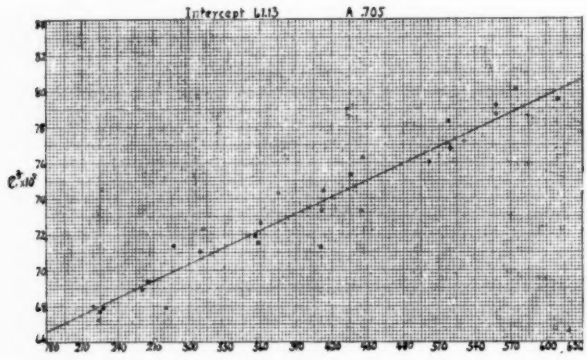


Fig. 3.

In conclusion I wish to express my thanks to Professor R. A. Millikan for suggesting this problem and for his kindly advice during the investigation, and also to Professor H. G. Gale for his timely suggestions. To my wife, also, I wish to express my appreciation for her assistance in the experimental part of the work.

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# THE OPTICAL PROPERTIES OF RUBIDIUM.

BY J. B. NATHANSON.

SOME time ago there appeared in the Astrophysical Journal,[1] an account of an investigation, I made, on the reflecting powers of sodium, potassium and rubidium. A direct method was used, employing a photo-electric cell as a photometer.

Up to the present, the polarimetric method of investigating the optical properties of the alkali metals, has been applied only to sodium by Paul Drude,[2] and to sodium and potassium by R. W. and R. C. Duncan.[3] It therefore seemed desirable to apply the polarimetric method to the determination of the optical properties of rubidium, at the same time affording a comparison between the values of the reflecting powers of rubidium as obtained by the former direct method, and the present polarimetric method.

## THE MIRROR.

The rubidium mirror used in this investigation was the same as that used in the former one, the mirror still being in very good condition. A description of the method of preparation of the mirror was given in The Astrophysical Journal, but for the sake of clearness it will be briefly repeated. The mirror was prepared in a vacuum by the distillation of the rubidium upon a piece of plane parallel glass *P* (Fig. 1), 2.5 cm. square, and 1.74 mm. thick. This glass plate formed part of a glass cell *C*. A mixture of rubidium chloride and calcium was placed in the hard glass tube *D*. Upon heating to a high temperature, the rubidium vapor passed to *A*, where it was condensed. The metal was purified by being redistilled from *A* to *B*. A small globule of the molten metal was then transferred to *F*, from where on further heating the metal was vaporized and condensed upon the glass plate *P*, the outside of which was kept ice cold. After the formation of the mirror, the cell was sealed off at *E*. Of several mirrors made, the best one was used in this investigation.

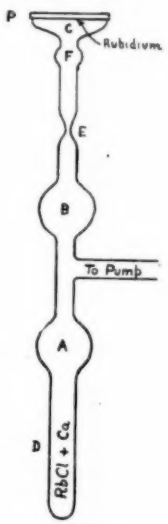


Fig. 1.

[1] Astrophysical Journal, 44, 137, 1916.
[2] Annalen der Physik, 64, 159, 1898.
[3] PHYS. REV., 36, 294, 1913.

## THE EXPERIMENTAL METHOD.

The optical constants were evaluated from observed values of the phase difference and azimuth of the reflected elliptically polarized light. These were determined by means of a simple Babinet compensator and two nicols mounted on a large spectrometer of the Societe Genevoise.

A 250-watt nitrogen-filled tungsten lamp was used as a source of light. One filament of this lamp was focused on the slit of a Hilger spectrometer $H$, Fig. 2. The eyepiece was removed, allowing a very narrow beam of monochromatic light to fall upon the slit of the collimator $C$. The beam of parallel rays then passed through the nicol $N_1$, whose plane of polarization was at an angle of 45° with the plane of incidence.

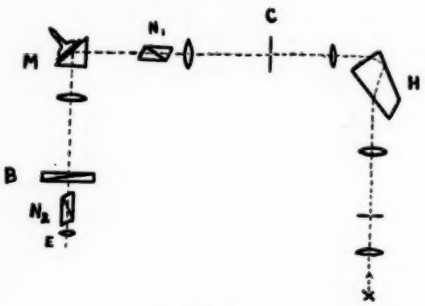



Fig. 2.

In order to avoid the disturbing reflection from the glass surface of the mirror $M$, the latter was pressed against the hypotenuse side of a right angle prism, cedar oil being placed between the prism and mirror. Light incident on one leg of the right angle prism was reflected from the mirror at an angle of 45°, passing out normally through the other leg of the prism. Thus the only changes in azimuth and in phase difference were those due to reflection at the metal glass boundary.

After reflection from the mirror, the elliptically polarized light was rendered plane polarized by the Babinet compensator $B$, and extinguished by the analyzing nicol $N_2$ which was viewed by the eyepiece $E$.

## METHOD OF OBSERVATION.

The constant of the Babinet compensator was determined several times for each wave-length used. Settings were made on the band of zero phase difference, and then on the bands of $-2\pi$ and $+2\pi$, there being ten readings taken for each position. The mean value of the constant for any wave-length was calculated from as many as 180 individual settings. Having obtained the position of the dark band representing zero phase difference, the telescope carrying the compensator and analyzing nicol was rotated through 90°, the mirror put in place, and the new position of the band noted. The amount of displacement of the dark band represents the phase difference $\Delta$ produced on reflection at the rubidium surface.

In order to obtain the azimuth $\psi$, the polarizing nicol was set with its

plane of polarization making successively angles of 45°, 135°, 225°, and 315° with the plane of incidence. For each position of the polarizer, the two positions of the analyzing nicol were determined by setting for maximum blackness of the bands. Ten readings were taken for each position, or a total of 80 settings for the determination of $\psi$. The mean of all the readings of the analyzer for two positions of the polarizer 180° apart was subtracted from the corresponding mean of all the readings for the other two positions of the polarizer. This difference is equal to $2\psi$.

The following example will illustrate briefly the method of calculating $2\psi$ and $\Delta$.

TABLE I.

$\lambda = 454.6\ \mu\mu$.

| Position of Polarizer. | 0° 18′. | 180° 18′. | 90° 18′. | 270° 18′. |
|---|---|---|---|---|
| Position of analyzer | 237° 42′<br>54° 36′ | 237° 42′<br>54° 54′ | 141° 48′<br>323° 54′ | 141° 54′<br>323° 30′ |
| Mean | 146° 9′ | 146° 18′ | 232° 51′ | 232° 42′ |
| Mean of means | 146° 14′ | | 232° 47′ | |
| Difference = $2\psi$ | 86° 33′ | | | |

*For Babinet Constant.*

| | $-2\pi$. | 0. | $+2\pi$. |
|---|---|---|---|
| Position of compensator | 20.270 | 27.581 | 34.839 |
| Differences | 7.311 | | 7.258 |
| Mean | 7.285 | | |

Position of compensator upon reflection from rubidium = 29.613

$$\Delta = \frac{29.613 - 27.581}{7.285} \times 360 = 100° 25'.$$

Attention must be called to the use of the right angle prism in eliminating disturbing reflections from the front of the mirror. Considerable trouble in the determination of $\Delta$ was experienced with the first prism used. It was found when studying the reflection from the prism itself, that the value of $\Delta$ obtained for internal reflection was about 30 per cent. less than the theoretical value of $\Delta$ as given by Drude's equation,

$$\tan\frac{\Delta}{2} = \frac{1}{n}\sqrt{n^2 - 2}, \qquad (1)$$

where the angle of incidence is 45°, and $n$ is the index of refraction of the glass prism. No amount of cleaning of the prism altered the value of $\Delta$. It was accordingly assumed that this deficit in $\Delta$ was due to internal strains, and so another prism was finally obtained which upon close examination yielded values of $\Delta$ agreeing to within one per cent. of the theoretical value.

## FORMULÆ.

Drude's equations in the rigorous form were used. The approximative equations as used for ordinary metals cannot be employed in this case due to the low value of the index of refraction, *i. e.*, the square of the sine of the angle of incidence cannot be neglected in comparison with the complex dielectric constant.

Let

$$\tan Q = \sin \Delta \tan 2\psi,$$
$$\cos 2P = \cos \Delta \sin 2\psi,$$
$$S = \sin \phi \tan \phi \tan P,$$

where $\phi$ is the angle of incidence = 45° throughout this investigation. The coefficient of absorption $k$ is given by

$$k = \tan \frac{X}{2}, \tag{2}$$

where

$$X = \frac{S^2 \sin 2Q}{S^2 \cos 2Q + \sin^2 \phi}.$$

The index of refraction $n$ is given by

$$n^2 = \frac{S^2 \cos 2Q + \sin^2 \phi}{1 - k^2}. \tag{3}$$

The principal angles of incidence and of azimuth are evaluated by means of the following equations:

$$\sin^4 \overline{\phi} \tan^4 \overline{\phi} = n^4(1 + k^2)^2 - 2n^2(1 - k^2) \sin^2 \overline{\phi} + \sin^4 \overline{\phi}, \tag{4}$$

$$k = \tan 2\overline{\psi}. \tag{5}$$

The reflecting power $R$ of the metal for normal incidence is given by

$$R = \frac{n^2(1 + k^2) - 2n + 1}{n^2(1 + k^2) + 2n + 1}. \tag{6}$$

## RESULTS.

The values of $\Delta$ and $2\psi$ and of the calculated optical constants are given in Table II. The values of $\Delta$ and $2\psi$ are the results of an extended

series of observations. The constants refer to the metal in contact with glass. The values of the reflecting powers as obtained directly by the use of the photo-electric cell are listed in the last column to afford comparison with those obtained by calculation from Drude's formulæ.

TABLE II.

*Metal—Glass Boundary.*

| λ in μμ. | Δ. | 2ψ. | $n$. | $k$. | $R$ (Calc.). | $R$ (Direct.) |
|---|---|---|---|---|---|---|
| 640.9 | 119° 30′ | 86° 52′ | 0.093 | 10.51 | 0.827 | 0.840 |
| 589.3 | 113 23 | 86 46 | 0.087 | 9.28 | 0.810 | 0.808 |
| 539.6 | 110 41 | 86 29 | 0.093 | 7.97 | 0.787 | 0.817 |
| 488.8 | 104 34 | 86 33 | 0.089 | 6.49 | 0.766 | 0.816 |
| 454.6 | 100 36 | 86 38 | 0.091 | 5.28 | 0.745 | 0.789 |

In general the reflecting powers as obtained directly by use of the photo-electric cell are somewhat lower than those obtained by the polarimetric method. It is possible that this may have been due to a slight deterioration of the mirror surface. It is also interesting to note that with the polarimetric method, the reflecting powers decrease more rapidly for the smaller wave-lengths than is the case with the photo-electric cell method. A comparison of the curves for the reflecting power is shown in Fig. 3.

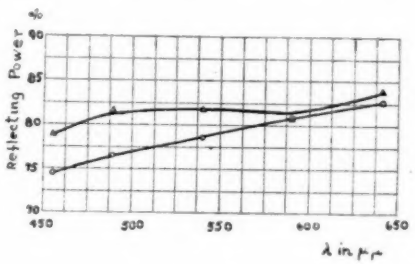



Fig. 3.

The principal angles of azimuth $\overline{\psi}$ and of incidence $\overline{\phi}$ can be calculated from the values of $k$ and $n$. In this case the values of $n$ in Table II. must be multiplied by the refractive index 1.51 of the glass plate of the mirror, in order to obtain the values of $n$ referring to the metal in contact with the air. It is assumed that $k$ remains the same. The results of the calculations are given in Table III., together with the reflecting powers for the metal-air boundary. There is very little variation in $n$.

TABLE III.

*Metal—Air Boundary.*

| λ in μμ. | $n$. | $\overline{\psi}$. | $\overline{\phi}$. | $R$ (calc.). |
|---|---|---|---|---|
| 640.9 | 0.140 | 42° 17′ | 62° 42′ | 0.840 |
| 589.3 | 0.131 | 41 56 | 60 1 | 0.811 |
| 539.6 | 0.140 | 41 26 | 58 44 | 0.780 |
| 488.8 | 0.134 | 40 37 | 55 29 | 0.739 |
| 454.6 | 0.137 | 39 39 | 53 22 | 0.700 |

From Tables II. and III. it appears that for the larger wave-lengths the reflecting powers for the air-metal boundary are greater than the reflecting powers for the metal-glass boundary, as we should expect. Oddly enough this is however reversed for the smaller wave-lengths.

Examination of R. W. and R. C. Duncan's[1] results for potassium, reveals a parallel case. For $\lambda = 665.0\ \mu\mu$ and $589.3\ \mu\mu$, the reflecting powers of potassium for the glass-metal boundary are less than for the air-metal boundary, while for $\lambda = 472.0\ \mu\mu$, the case is just reversed, *i. e.*, $R$ (air-$K$) = 86.9 per cent. while $R$ (glass-$K$) = 87.8 per cent. It follows that for some value of the wave-length, the reflecting power of the metal must be the same irrespective of whether there is air or glass as the medium in contact with the metal. If this is the case, then

$$\frac{n^2(1+k^2)-2n+1}{n^2(1+k^2)+2n+1} = \frac{n^2(1+k^2)1.51^2-2n\cdot 1.51+1}{n^2(1+k^2)1.51^2+2n\cdot 1.51+1},$$

where 1.51 is the index of refraction of the glass, and $n$ refers to the glass-metal boundary. Solving this equation for $k$,

$$k = \frac{1}{n}\sqrt{0.66-n^2}. \qquad (7)$$

For large values of $k$ and small values of $n$, *i. e.*, $n < 0.81$, the right-hand side of the equation is real, and the equality is possible.

The calculations for the reflecting powers have been made on the assumption that $k$ of the metal is not affected by the character of the medium in contact with that metal. Various investigations on this point do not seem to be in harmony. Ingersoll[2] showed experimentally that the reflecting power of a metal in contact with air can be obtained from the values of $n$ and $k$ for the metal in contact with a transparent medium, by multiplying $n$ by the refractive index of that medium, and assuming $k$ unchanged. On the other hand Tate's[3] results for silver in contact with air and with glass, show that $k$ as well as $n$ is affected by the medium in contact with the metal, $k$ being about half as large for the silver-glass boundary as for the silver-air boundary. In fact Tate's values for the reflecting powers of silver in contact with air cannot be obtained from the values of $n$ and $k$ for the silver-glass boundary, by merely multiplying $n$ by 1.51 and keeping $k$ constant.

From all the aforesaid, it therefore appears unreliable to calculate the reflecting power of an air-metal boundary from the values of $n$ and $k$

[1] PHYS. REV., 36, 294, 1913.
[2] PHYS. REV., 29, 392, 1909.
[3] PHYS. REV., 34, 327, 1912.

obtained from the metal-glass boundary, by merely correcting for $n$. It is not safe to assume that $k$ remains the same. Further investigation of this question is desirable.

## SUMMARY.

The optical constants of rubidium were obtained for wave lengths ranging from 454.6 $\mu\mu$ to 640.9 $\mu\mu$. A simple Babinet compensator and two nicols were employed to measure the phase difference and azimuth. The constants were calculated by means of Drude's formulæ.

The rubidium mirror was formed by distillation of the metal in vacua, with subsequent condensation upon a piece of plane parallel glass. A right angle prism served to eliminate troublesome reflections from the glass front of the mirror.

The reflecting powers of the metal in contact with glass were, with the exception of that for $\lambda = 589.3$ $\mu\mu$, somewhat lower than those obtained directly by means of a photo-electric cell in a previous investigation.

The results do not warrant the assumption that the coefficient of absorption of rubidium remains constant irrespective of the medium in contact with the metal.

CARNEGIE INSTITUTE OF TECHNOLOGY,
PITTSBURGH, PA.,
October, 1917.

# THE THEORY OF IONIZATION BY COLLISION.

## IV. CASES OF ELASTIC AND PARTIALLY ELASTIC IMPACT.

BY K. T. COMPTON AND J. M. BENADE.

*Introduction.*—In previous papers by one of the writers[1] a theory was developed by which the rate of ionization of molecules of a gas at pressure $p$ by electrons moving in a field of intensity $X$ could be calculated in two particular cases, viz., if the collisions of electrons with molecules are inelastic and if the collisions other than ionizing collisions result in no loss of energy. In the latter case, which was called "the case of elastic impact," it was shown that the average number of ionizing collisions $\alpha$ made by an electron while advancing one centimeter bears to the pressure $p$ and the intensity $X$ the relation

$$\frac{\alpha}{p} = \psi\left(\frac{X}{p}\right), \tag{1}$$

in which the form of the function $\psi$ is determined by

$$\alpha = P\nu = pN\sqrt{P} \tag{2}$$

and

$$\epsilon^{P\nu x_0} = \frac{2}{1+P}\left\{\frac{1}{[\nu x_0(1+P)]} + \frac{2(\nu x_0 - 1)}{[\nu x_0(1+P)]^2} + \frac{3(\nu x_0 - 1)(\nu x_0 - 2)}{[\nu x_0(1+P)]^3} + \cdots + \frac{(\nu x_0 - 1)(\nu x_0 - 1)!}{[\nu x_0(1+P)]^{\nu x_0 - 1}} + \frac{\nu x_0!}{[\nu x_0(1+P)]^{\nu x_0}}\right\}. \tag{3}$$

In these equations $P$ is the probability of ionization at a collision; $\nu$ is the average number of collisions made by an electron while advancing one centimeter toward the anode; $pN$ is the average number of collisions made by an electron while moving one centimeter in its actual zig-zag path; $N$ is this quantity calculated for 1 mm. pressure, and is the reciprocal of the mean free path at 1 mm. pressure; $x_0 = V_0/X$, where $V_0$ is the minimum ionizing potential. These equations were found to agree well with experimental determinations of $\alpha$ in helium[2] when the constants $V_0$ and $N$ were given values differing very little from accepted experimental values. The small discrepancy between theory and experiment was attributed to impurities in the helium.

[1] PHYS. REV., 7, pp. 489, 501, 509, 1916.

[2] Phil. Mag., 23, p. 837, 1912.

Two lines of evidence, however, have recently indicated that the assumptions underlying equations (1), (2) and (3) must be modified if they are to be applied to helium and similar gases, and have suggested the nature of this modification. The first of these is the fact that, even though collisions in helium are perfectly elastic, yet sufficient energy is transferred from the electron to the molecule at impact to affect appreciably the rate of ionization of the gas. A detailed study of this loss of energy has recently been published by the writers.[1] The second line of evidence is based on the following study of Stoletow's constant.

*Stoletow's Constant.*—It has been shown by Townsend[2] that, if there is a functional relation of the type

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right),$$

it necessarily follows that the ratio of the intensity $X$ to the pressure $p_m$ at which $\alpha$ is a maximum is constant for all values of $X$. This ratio $X/p_m$, whose value is characteristic of the gas, is Stoletow's constant and has been measured and verified in the case of a number of gases of the inelastic type.

If there were a gas in which electrons lose no energy at impacts, except in the process of ionization, it is obvious that for such a gas $p_m$ would be infinite and Stoletow's constant $X/p_m$ would equal zero. The following experiments were made to test this point in the case of helium.

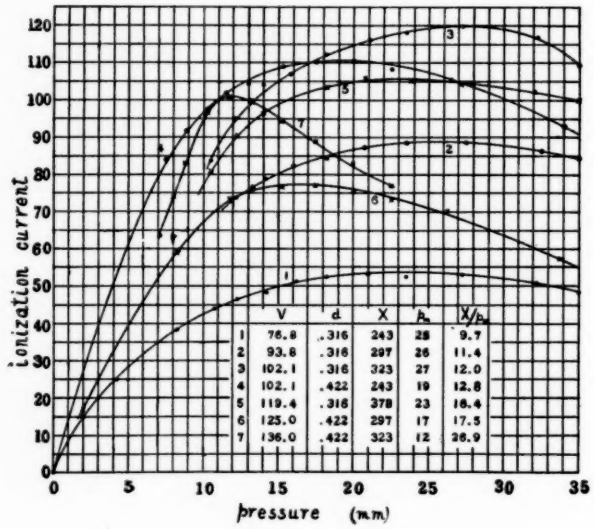
Carefully purified helium was introduced at various pressures into an ionization chamber containing two parallel electrodes. From one of these, electrons were liberated by ultra-violet light and moved under the influence of the applied field to the second electrode, which was connected to an electrometer, shunted with a high resistance. The details of the purification of the helium and the construction of the apparatus have been described in an earlier paper. The experimental procedure was to vary the pressure, keeping other conditions constant, until the pressure was discovered at which the current through the gas was maximum. A small correction of these results was necessary to take account of the regular decrease of photoelectric emission from the cathode as the pressure was increased. This correction was easily determined by a control experiment. Fig. 1 shows the result of a number of such tests with various values of the field $X$ and the distance $d$ between the electrodes.

It is very evident that $X/p_m$ cannot be considered constant. That this

[1] PHYS. REV., 10, pp. 77, 80, 1917.

[2] Electricity in Gases, p. 300.

lack of constancy is not due to insufficient purity of the helium is proven by our previously reported measurements of the elasticity of impact in this same helium. It is necessary to conclude, therefore, that the functional relation of equation (1) is not true in the case of helium, which




| | V | d | X | $p_c$ | $X/p_c$ |
|---|---|---|---|---|---|
| 1 | 76.8 | .316 | 243 | 25 | 9.7 |
| 2 | 93.8 | .316 | 297 | 26 | 11.4 |
| 3 | 102.1 | .316 | 323 | 27 | 12.0 |
| 4 | 102.1 | .422 | 243 | 19 | 12.8 |
| 5 | 119.4 | .316 | 378 | 23 | 16.4 |
| 6 | 125.0 | .422 | 297 | 17 | 17.5 |
| 7 | 136.0 | .422 | 323 | 12 | 26.9 |

Fig. 1.

proves that the energy lost at non-ionizing collisions in helium cannot be neglected.

The following treatment of the theory takes account of small energy losses at collisions and should be applicable to all cases of elastic and nearly elastic impact.

*Theory.*—Let $\Delta e$, where $e$ is the charge on an electron, represent the average amount of energy lost by an electron at a non-ionizing collision. Then, of the energy $Xe$ acquired from the field while advancing 1 cm., an electron loses on the average an amount $\nu\Delta e$ by these collisions. Thus, if $X'e$ represents the *net* gain of energy per centimeter, we have

$$X'e = Xe - \nu\Delta e. \tag{4}$$

Obviously, if we insert $X'$ in place of the actual intensity $X$ in equations (1) and (3), we take account of losses of energy at non-ionizing collisions. Thus equation (1) in its general form should be written

$$\frac{\alpha}{p} = \psi\left(\frac{X'}{p}\right) = \psi\left(\frac{X - \nu\Delta}{p}\right). \tag{5}$$

In case there is no loss of energy except in ionization, $\Delta = 0$ and equation (5) reduces to equation (1). In case collisions are entirely inelastic, $\Delta$ is proportional to $X$ and equation (5) reduces to Townsend's relation

$$\frac{\alpha}{p} = f\left(\frac{X}{p}\right),$$

in which form of the function $f$ has been discussed in preceding papers. For cases in which collisions are nearly or entirely elastic, it is evident that $\Delta$ depends on the maximum energy $V_0 e$ acquired and not appreciably on the field $X$. We shall proceed to develop equation (5) into a form applicable to experimental measurements in gases of this latter type.

It was shown in an earlier paper[1] that

$$\nu = \frac{\frac{1}{2}mv^2p^2N^2}{Xe},$$

where $v$ is the average velocity of an electron just before it ionizes. From equation (4),

$$\frac{1}{\nu} = \frac{X'e + \nu\Delta e}{\frac{1}{2}mv^2p^2N^2}.$$

But

$$\frac{X'e}{\frac{1}{2}mv^2} = \alpha = P\nu$$

is the average number of times an electron ionizes while advancing one centimeter, while

$$\frac{\Delta e}{\frac{1}{2}mv^2} = \delta,$$

where $\delta$ is the ratio of the average energy lost at a non-ionizing collision to that lost at an ionizing collision. Thus

$$\frac{p^2N^2}{\nu^2} = (P + \delta). \tag{6}$$

Eliminating $P$ by the relation $P\nu = \alpha$, and solving for $\nu$ we obtain

$$\nu = \frac{2N^2p}{\frac{\alpha}{p} + \sqrt{\frac{\alpha^2}{p^2} + 4N^2\delta}}. \tag{7}$$

If we substitute this value of $\nu$ in equation (5) we obtain the relation

$$\frac{\alpha}{p} = \psi\left(\frac{X'}{p}\right) = \psi\left(\frac{X}{p} - \frac{2\Delta N^2}{\frac{\alpha}{p} + \sqrt{\frac{\alpha^2}{p^2} + 4N^2\delta}}\right), \tag{8}$$

[1] PHYS. REV., 7, p. 510, 1916.

which is in a form suitable for experimental test. The most convenient method of handling experimental data is to substitute the observed values of $X/p$ in equation (8) and calculate $X'/p$. Equations (2) and (3) are then directly applicable if we put $x_0 = V_0/X'$.

Equation (3) has been solved for $P$ corresponding to the values of $\nu x_0$ given in Table I. Corresponding values of $pNV_0/X'$ are determined by use of equation (2). Intermediate values may be determined graphically.

TABLE I.

| $\nu x_0$. | $P$. | $\frac{pNV_0}{X'}$. | $\nu x_0$. | $P$. | $\frac{pNV_0}{X'}$. |
|---|---|---|---|---|---|
| 1 | 0.2490 | 0.499 | 20 | 0.0265 | 3.255 |
| 2 | 0.1610 | 0.802 | 30 | 0.0186 | 4.090 |
| 3 | 0.1213 | 1.045 | 40 | 0.01436 | 4.793 |
| 4 | 0.0984 | 1.255 | 60 | 0.00989 | 5.965 |
| 5 | 0.0831 | 1.441 | 80 | 0.00755 | 6.950 |
| 6 | 0.0723 | 1.613 | 100 | 0.00613 | 7.830 |
| 8 | 0.0574 | 1.917 | 150 | 0.004175 | 9.695 |
| 10 | 0.0478 | 2.186 | 200 | 0.003175 | 11.270 |
| 15 | 0.0340 | 2.768 | 275 | 0.002332 | 13.300 |

*Comparison with Experiment. Helium.*—The values of $X/p$ and $\alpha/p$ in Table II. were determined experimentally by Gill and Pidduck,[1] and the values of $X'/p$ were calculated by equation (8). To do this $V_0$ and $N$ were chosen to give the best agreement between theory and experiment; $\Delta$ was taken to be the energy lost at an impact by an electron moving with half the ionizing energy, and is known with considerable accuracy as a result of our recent measurements of energy losses; $\delta = \Delta/V_0$.

TABLE II.

$V_0 = 21$ volts. $N = 8.7$.
$\Delta = 0.00282$ volts. $\delta = 0.000134$.

| $\frac{X}{p}$. | $\frac{\alpha}{p}$. | $\frac{X'}{p}$. |
|---|---|---|
| 5.0 | 0.127 | 3.83 |
| 10.0 | 0.275 | 9.31 |
| 10.0 | 0.285 | 9.33 |
| 20.0 | 0.560 | 19.63 |
| 20.0 | 0.597 | 19.65 |
| 38.1 | 1.035 | 37.90 |
| 40.0 | 1.080 | 39.80 |
| 80.0 | 1.835 | 79.90 |
| 120.0 | 2.100 | 120.0 |
| 200.0 | 2.370 | 200.0 |

[1] Phil. Mag., 23, p. 837, 1912.

The last two sets of observations are not plotted, since the experimental conditions under which they were taken have been shown in an earlier paper to be misleading.

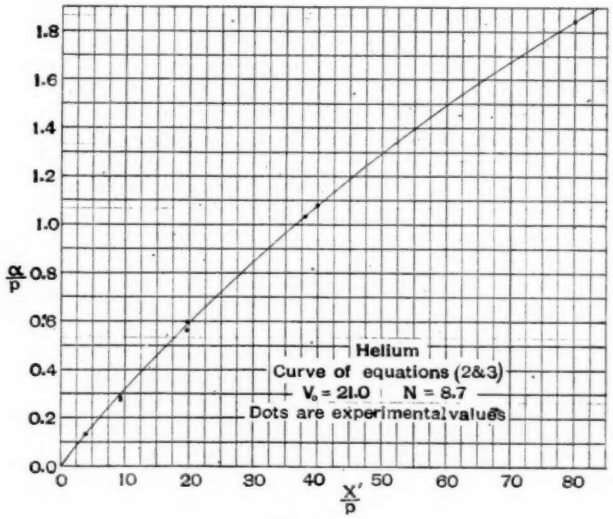



Fig. 2.

The remarkable agreement between theory and experiment is shown by Fig. 2, in which the solid curve represents equations (2) and (3) and the dots represent the observations in Table II. The discrepancies are certainly within the limits of experimental error.

Further support of the theory is afforded by the values of $V_0$ and $N$, which are the parameters of the equations. Probably the minimum ionizing potential is nearer 20 volts than 21 volts, but 21 volts is within the range of accepted direct measurements. It is not so easy to decide on the correct value of $N$, since we estimate $N$ from considerations based on the kinetic theory of gases, and it is not certain that the effective molecular cross section which functions in collisions of molecules with each other is pertinent to the present problem. Assuming that it is, however, we find values ranging from $N = 8.3$ to $N = 13.5$, depending on the method of calculation. The smaller values result from taking the electronic free path to be $4\sqrt{2}$ times that of a gas molecule and the larger values from $N = \pi r^2 n$, where $r$ is the molecular radius and $n$ the number of molecules per unit volume. The former method of calculation has been more widely accepted, and there is no reason, therefore, for doubting the accuracy of the value $N = 8.7$.

*The Case of Hydrogen.*—It is supposed that impacts in hydrogen are more elastic than those in other gases, with the exception of the mona-

tomic gases. This view is supported by rough measurements by Franck and Hertz[1] of the average energy lost at a collision and by attempts to apply equations for inelastic impact to the case of ionization in hydrogen. For instance, if an attempt is made to fit the equation for inelastic impact developed by one of the writers[2] to the experimental data published by Townsend[3] and Townsend and Hurst,[4] good agreement is obtained if the minimum ionizing potential is taken to be $V_0 = 9.56$ volts. In dealing with all other gases the equation leads to values of $V_0$ which are too large, while in this case it leads to a value which is distinctly too small. The most probable explanation of this discrepancy is that electrons retain some energy after non-ionizing impacts. The equations of this paper, however, are much less successful than those of inelastic impact. This supports the evidence, which we have advanced in our former papers, that losses of energy at impacts in hydrogen are due to processes similar to those which are effective in the so-called inelastic gases, and which are typically different from those which produce energy losses in gases like helium. The energy lost in inelastic gases, we believe, appears as energy of vibration of parts of the molecular complex.

*Discussion.*—The equations developed in this paper should be, and appear to be, more accurate than any that have been proposed for the case of elastic impact. The reason for this lies in the fact that all such equations must be based on some assumption regarding the probability that an electron, whose energy is greater than the minimum ionizing energy, will ionize at a collision. Until the mechanism of ionization is better understood, the expressions suggested for this probability must be entirely empirical and the best of them is probably only an approximation to the truth. Any error in the form of this expression, however, affects the accuracy of equations for elastic impact much less than those for inelastic impact. For if an electron, possessing at least the minimum ionizing energy, fails to ionize at an inelastic impact it loses its chance until it has gathered a new supply of energy; while if it fails to ionize at an elastic collision it retains its ability to ionize at the next collision. Since collisions are comparatively numerous in elastic gases, this means that an electron advances very little beyond the point at which it has accumulated the ionizing energy until it ionizes. There is reason, therefore, for confidence in equations (2), (3) and (8).

PALMER PHYSICAL LABORATORY,
PRINCETON, N. J.

[1] Verh. d. D. Phys. Ges., 15, p. 373, 1913.
[2] Loc. cit.
[3] Phil. Mag., 6, p. 598, 1903.
[4] Ibid., 8, p. 738, 1904.

PROCEEDINGS

OF THE

# AMERICAN PHYSICAL SOCIETY.

## VACUUM GAUGES OF THE RADIOMETER TYPE.[1]

BY R. G. SHERWOOD.

A VACUUM gauge, based on the principle of molecular bombardment, was designed in 1910 by M. Knudsen.[2] Woodrow[3] modified the design to remove some of its limitations. By making further modifications in construction, Mr. J. E. Shrader and myself at the Westinghouse Research Laboratory produced a gauge of simple construction capable of measuring pressures as low as $10^{-8}$ mm. of Hg., possessing good stability and not expensive to build.

The theory of the gauge as derived by M. Knudsen makes this gauge applicable as an absolute instrument only at comparatively low pressures. It is desirable to extend the range well up into that covered by a mercury manometer. This may be done by making the proper corrections for molecular collisions and for unbalanced impacts.

The principle involved in the operation of this type of gauge is that of molecular bombardment. Molecules of gas leaving a platinum strip, heated electrically, bombard a suspended vane hung parallel and close to the platinum strip causing it to turn. If the distance between the movable vane and the platinum strip is small compared with the mean free path of the gas molecules and the dimensions of the vane and strip such that the edge effect can be neglected then Knudsen has shown that the following formula holds:

$$P = \frac{2FT_2^{1/2}}{T_1^{1/2} - T_2^{1/2}}, \tag{1}$$

where $T_2$ = temperature absolute in gas without vanes, $T_1$ = temperature absolute of heated platinum strip, and $F$ = force of molecular repulsion. For temperature differences not greater than 250° C. the formula holds well if written

$$P = \frac{4FT_2}{T_1 - T_2}. \tag{2}$$

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

[2] Ann. d. Phys., IV., 32, 809, 1910; 44, 525, 1914.

[3] PHYS. REV., IV., 6, 491, 1914.

If $R_1$ = electrical resistance of platinum strip when heated,
$R_2$ = electrical resistance at temperature of gauge,
$K$ = constant of the gauge.
$S$ = scale reading.

Then as a working formula (2) reduces to

$$P = \frac{KR_2}{R_1 - R_2}. \tag{3}$$

For the dimensions of the elements in the gauge designed for laboratory use, pressure above $10^{-4}$ mm. of the Hg cannot be measured with any degree of precision from the above formulæ without correcting for

(1) Collisions,
(2) Edge effect.

The following formula has been found to give the necessary corrections for one of these gauges up to 0.05 mm. of Hg on air.

$$P = \frac{\lambda}{d}\left(e^{\frac{d}{\lambda}} - 1\right) c \left(\frac{K \cdot S \cdot R_2}{R_1 - R_2}\right), \tag{4}$$

where $\lambda$ = mean free path of the air molecules, $d$ = distance between the movable vane and Pt heating strip. $c$ = constant depending upon the ratio of the width of the suspended vane to $d$; the length of the vane being large as compared to the other dimensions is not considered in deriving the expression for the above correction, $e$ = base of natural logarithms.

For greater pressures up to 1 or 2 cm. of Hg the gauge makes an ideal detector for small changes in pressure, but in its present form is not suitable for absolute measurements.

The sensibility of the gauge increases from nearly zero at about 2 cm. Hg to a maximum at about 0.05 mm. Hg; then decreases toward zero for very low pressures.

WESTINGHOUSE RESEARCH LABORATORY,
E. PITTSBURGH, PA.

## FURTHER VERIFICATION OF KNUDSEN'S EQUATIONS FOR RESISTANCE TO MOLECULAR FLOW.[1]

BY L. E. DODD.

KNUDSEN[2] has developed from the kinetic theory of gases expressions for the "resistances" of an aperture and of a tube of given dimensions, to the passage of gas molecules under conditions of sufficiently low pressure that the molecular collisions occur in relatively small number and the flow is

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

[2] Knudsen, Annalen der Physik, 28, p. 75, also p. 1009, 1909.

thus purely "molecular." In the case of an aperture the resistance is given by $W_1 = 2\pi/\sqrt{A}$, where $A$ is area of aperture. For a tube the resistance is $W_2 = 3/8\sqrt{\pi/2}\int_0^L o/A^2\,dl$, where $L$ is length of tube, o is circumference, and $A$ cross-sectional area. Knudsen verified these expressions experimentally with hydrogen, oxygen, and $CO_2$. By their practical use he determined the vapor tension of mercury over the temperature range from 890° down to − 50° C.

Egerton[1] in England, working on the vapor tensions of zinc and cadmium, has further verified the equations with mercury as the standardizing material for his tubes. He used not only a single aperture but as many as seventeen apertures in parallel in the same partition, finding that in the case of two or more apertures of equal area in parallel the total resistance is obtained by dividing the resistance for one aperture by the number of apertures.

In connection with work on the vapor tension of selenium the writer has found that Knudsen's expressions for resistance to molecular flow are experimentally valid. Preliminary to the measurements on selenium it was thought desirable to standardize two tubes with mercury. The tubes were similar to those employed by Egerton. One of them had two apertures in parallel, and the other tube six apertures. With the tube having the two apertures the mean value of sixteen separate determinations of the resistance (due both to apertures and the portion of the tube lying between apertures and region of condensation) agreed well with the value as predicted from the Knudsen equations. The per cent. of probable error from the mean was less than two, which was regarded as satisfactory. With the other tube the equations also hold, at least approximately.

State University of Iowa,
Iowa City, Ia.

## Rectification of Alternating Current by the Corona.[2]

By J. W. Davis.

It has been possible to rectify voltages as high as 42,000 volts effective by means of the corona discharge in hydrogen. The rectification is practically perfect, but the efficiency is not very high, as a large amount of energy is wasted in the discharge itself. For a given gas pressure the maximum voltage which may be rectified is approximately directly proportional to the radius of the outer cylinder, when the inner cylinder is small compared with the outer cylinder. An incandescent wire will give a croona discharge at voltages much lower than those necessary to start a discharge from a cold wire. The heat

[1] Capt. A. C. Egerton, Philosophical Magazine, 33, p. 33, Jan., 1917.

[2] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

conductivity of a gas is largely increased in regions where ionization by collision takes place.

UNIVERSITY OF ILLINOIS,
URBANA, ILLINOIS,
November 15, 1917.

## A MONO-WAVE-LENGTH X-RAY CONCENTRATOR.[1]

BY ELMER DERSHEM.

THE derivation is given of the mathematical equation of a curved surface which may be used to concentrate X-rays of a single wave-length by reflection from bent mica crystals placed on this surface. This surface is shown to be a logarithmic spiral surface of revolution.

Methods of constructing such a surface and experimental results are given showing that it is possible by this means to concentrate upon a small area X-rays of a single frequency a thousand times as intense as can be obtained by reflection from plane crystals.

STATE UNIVERSITY OF IOWA.

## WAVE-LENGTHS OF THE TUNGSTEN X-RAY SPECTRUM.[1]

BY ELMER DERSHEM.

THE factors affecting the resolving power of an X-ray spectrometer are discussed and it is shown that for precise measurements a thin crystal must be used and corrections made for the width of the source.

Experimental results are given showing that the L group of the tungsten X-ray lines contains at least 19 lines and precise values of their wave-lengths as well as those of the four K lines of tungsten are given. These results are compared with those of other investigators.

STATE UNIVERSITY OF IOWA,
IOWA CITY, IA.

## A MEGAPHONE WITH A RECTANGULAR APERTURE.[1]

BY F. R. WATSON.

THE theory for such a horn has been given by Rayleigh.[2] When a train of parallel waves pass through a slit whose width is equal to or smaller than half the wave-length, they spread out as if they come from the aperture as a center of disturbance. When the width of the aperture is large compared with the half wave-length, there is but little spreading out and the waves proceed almost undisturbed. Using this conception, Rayleigh constructed a horn with an elliptical aperture, the major axis of the ellipse being large

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

[2] On the Production and Distribution of Sound, Phil. Mag., Vol. VI, pp. 289–305, 1903.

compared with the wave-length and the minor axis rather smaller than the half wave-length. If the major axis is held vertical and the axis of the horn is horizontal, the sound is spread out in a fan-shaped horizontal layer.

Rayleigh applied this horn experimentally in testing several properties of sound waves. He also suggested that similar horns of larger dimensions might be useful in fog signalling, a suggestion that was carried out with successful results.[1]

It occurred to the author that the principle might be extended to the important domain of speech sounds. With this object in view, several horns with rectangular apertures were constructed. the dimensions being varied in the different horns. These were tried in an open field and pronounced results were obtained. When the horn was held horizontally with the rectangular aperture vertical, the sounds were diffracted more than 90° from the straightforward direction, and observers off to the side heard easily. When, however, the horn was rotated 90°, so that the long edge of the rectangular opening was horizontal, the sound heard by the observers was indistinct. Several applications of the horn are suggested. It would be serviceable in making announcements to a crowd on bleachers, in directing the sounds from a phonograph, etc.

The results obtained bear on another point of some importance, namely, the mean wave-length of speech. The fact that speech sounds were diffracted by the narrow dimension of the aperture, but not by the large dimension indicates that some effective component of speech has a half wave-length lying between these two limits. This deduction is of value in estimating the dimensions of relief work on the walls of auditoriums, where it is desired to scatter the sound. It is also useful in telephony in adjusting the pitch of telephone plates to resound to the speech sounds.

UNIVERSITY OF ILLINOIS.

## A NEW HYDRATE OF URANIUM NITRATE; URANIUM NITRATE ICOSITETRAHYDRATE.[2]

BY FRANK E. E. GERMANN.

WHEN a water solution of uranium nitrate is cooled to − 180° C., it is possible to get various distinct fluorescent spectra from it, depending on the rate of cooling of the solution. H. L. Howes described what seemed to be five distinct spectra, varying from a sharp-lined spectrum in the case of slow cooling, to a broad-banded spectrum in the event that the solution was plunged directly into liquid air. The other spectra were the result of other chance intermediate methods.

It was while trying to find the cause of this unexplained action that the author discovered a hitherto undescribed hydrate of uranium nitrate, stable below about − 19° C., forming spontaneously at about − 35° C., at which

[1] Sound Signals, Soc. Arts Journal, Vol. 50, pp. 315–327, 1902.

[2] Abstract of a paper presented at the Rochester meeting of the American Physical Society, October 26 and 27, 1917.

point there is very rapid liberation of heat, warming up the specimen as much as 20° in some cases. Due to the phenomenon of supercooling, the temperature of formation, and the maximum temperature of stability have not yet been definitely fixed.

The composition of the new hydrate was determined by the method of thermal analysis, working with thermo-couples, and measuring the heats of formation from equal volumes of solutions of varying percentages of concentration. Neglecting the fact that the specific heats of different concentration solutions are not equal, and plotting heat liberated against percentage concentration, two straight lines resulted, cutting each other sharply at a percentage corresponding to 47.6 parts of anhydrous uranium nitrate, $UO_2(NO_3)_2$, to 100 parts of solution. The uranium nitrate hexahydrate, $UO_2(NO_3)_2 .6H_2O$, which is the stable nitrate at room temperatures, contains 78.5 per cent. anhydrous salt, whereas a percentage of 47.7 anhydrous salt corresponds to the formula $UO_2(NO_3)_2 .24H_2O$, which we may call *uranium nitrate icositetrahydrate*. Although a compound containing twenty-four molecules of water is by far the most probable, still it may be worthy of noting that a molecule containing twenty-three molecules of water would contain 48.8 per cent. salt, and one with twenty-five molecules would contain 46.7 per cent.

The transformation can be suppressed by rapid cooling, in which case it takes place on heating up. The formation is accompanied by a fairly large increase in volume, so that by cooling a specimen down in a very heavy glass tube, the transformation may be suspended, even when cooled slowly, but on warming up, the tube is usually shattered at the transformation point.

The formation of the icositetrahydrate explains in part the various spectra, but it does not explain the fact that a banded spectrum may be produced. This may be due to an amorphous condition of the solid mass, or to a possible solidification in the exact state in which it existed as a solution, in which state it normally gives a banded spectrum. The author will take up the latter consideration in a future article.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY,
ITHACA, N. Y.,
October 11, 1917.

## A CORRECTION IN THE THEORY OF IONIZATION BY COLLISION.[1]

BY JAKOB KUNZ.

IT has been shown by Bergen Davis, F. S. Goucher, Y. T. Tate, and P. D. Foote, that for the metallic vapors radiation is emitted without ionization when electrons collide with the atoms of the vapor, a radiation, which may give rise to a photoelectric effect on the electrodes and thus resemble an

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

ionization by collision. These "resonance" voltages are for

| | | |
|---|---|---|
| Hg : 4.9 volts, | Zn : 4.1 volts, | Cd : 3.88 volts. |
| Na : 2.10 volts, | K : 1.55 volts, | |

The ionizing potentials, however, are:

| | | |
|---|---|---|
| Hg : 10.4, | Zn : 9.5, | Cd : 8.92, |
| Na : 5.13, | K : 4.1. | |

Davis and Goucher found, moreover, that an increase in the intensity of radiation takes place at an impact voltage of about 6.7 volts in Hg vapor. The results obtained for H by these authors are quite different; ionization by impact and emission of radiation occur at 11 volts. A second type of ionization by collision without increase of radiation occurs at about 15.8 volts and a second type of radiation without an increase in ionization is emitted at 13.6 volts. In these measurements accelerating potential differences of 2 up to 20 volts have been used. In the experiments on ionization by collision by Townsend and others potential differences of 100 to 400 volts have been used. Nevertheless it has been assumed that the current only increased by the increase of the number of positive and negative ions through the process of collision. But the ultra-violet light, which in many cases will arise under the influence of the high potential differences, must contribute to the ionization. This may explain the fact that in many cases the saturation current is not constant but increases slightly. If we assume that the ultra-violet light is distributed uniformly through the ionization chamber and that it produces $n$ electrons per unit volume and per unit time, then we have in a layer of thickness $dx$ the number $n_1 dx$, which when moving through $x$ will produce $dn = n_1 dx\, e^{\alpha x}$ new electrons, and the number

$$n = \int_0^l n_1 e^{\alpha x} dx = \frac{n_1}{\alpha}(e^{\alpha l} - 1)$$

has to be added to the number of ions $n_1 = n_2 e^{\alpha l}$, which are produced by collision alone. If the light is not uniformly distributed between the two plates or if the electrodes are of different chemical material, a photoelectric effect of the plates must be considered in addition.

The existence of the resonance potential raises a very interesting question with respect to the photoelectric effect. For instance, in sodium vapor a potential difference of 2.10 volts is sufficient to produce yellow light according to $eV = hn$. If this yellow light were able to ionize the vapor, then we would have a photoelectric effect, and the resonance potential would not be distinguished from the ionization potential which is 5.13 volts, corresponding to ultra-violet light. These potential differences, if correct, indicate therefore that sodium vapor will show a photoelectric effect only for ultra-violet light, while sodium metal of course shows a photoelectric effect for visible light.

It looks therefore as if sodium vapor and sodium metal were very different with respect to the photoelectric effect. Measurements to decide this question are in progress in our laboratory.

UNIVERSITY OF ILLINOIS,
URBANA, ILL.

## MOBILITY OF IONS IN AIR, HYDROGEN, AND NITROGEN.[1]

BY KIA-LOK YEN.

THE primary aim of this experiment was to determine the mobility of the hydrogen and nitrogen ions by means of the high-frequency high-potential method employed by L. B. Loeb in his determination of the mobility of ions in air (see PHYS. REVIEW, N. S., Vol. VIII., No. 6, 1916, pp. 633–650). But, as the Loeb experiment was the only one of its kind that had ever been performed before, it was thought worth while to repeat it before extending it to other gases. Consequently, the mobility of ions in air was redetermined before the determination of the hydrogen and nitrogen ions by the same method.

When working with air, a field strength of as high as 14,160 volt/cm. was employed and neither the positive nor the negative ions exhibited any tendency to deviate from the law that the product of the mobility times the pressure is a constant. A potential of 6,668 volt/cm. was applied in the case of hydrogen and there was no indication whatsoever of any abnormal increase in either the positive or the negative mobility. For nitrogen, the potential employed was as high as 17,670 volt/cm. and yet not even the slightest deviation from the law, as stated above, was manifested by either the positive or the negative ions.

Free electrons were found in both hydrogen and nitrogen; more in the former than in the latter. It might be expected that with high potentials the appearance of electrons would be more favored, but in the experiments made to test this point, fewer electrons were found in hydrogen and nitrogen with high potentials than when the mobilities were measured with the ordinary 60 cy. low alternating potentials. This seems to fit in with the idea suggested by Wellisch that a certain speed is necessary before the negative electron can attach itself to a neutral molecule to form a negative ion.

Besides the free electrons and the normal negative ions no trace of any other kind of negative ions could be found.

Thus, an additional argument is offered by these results in favor of the small-ion theory as well as against the cluster hypothesis.

RYERSON LABORATORY,
UNIVERSITY OF CHICAGO,
CHICAGO, ILL.

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

## The Determination of Organic Compounds by an Optical Method.[1]

By Thos. E. Doubt and B. B. Freud.

THE index of refraction has long been recognized as one of the characteristics of a body. The index of refraction for a number of wave-lengths may be determined by a single photograph of Talbot's bands for a known thickness of the body. Gibbs has proved that for a number of common substances his interferential constant which is obtained by dividing the number of Talbot's bands between two spectrum lines by the density of the substance is independent of the temperature. He suggested that this new constant be used in determining the composition of mixtures. We have found no further application of this suggestion. A comparison of this constant with the other so-called constants; namely, Newton's, Gladstone and Dale, and Lorentz and Lorenz is made in the paper. The values of these constants for benzol and toluol are given in tables and it is seen that the values of the interferential constant differ by the greatest amount. Between the two given lines for a layer of liquid one centimeter thick there are 110 bands more in the case of toluol than in the case of benzol.

Armour Institute of Technology,
Chicago, Ill.

## The Analysis of Polarized Light Reflected from Small Opaque Crystals.[1]

By LeRoy D. Weld.

WHEN plane-polarized light falls upon a polished metal, it is in general elliptically polarized upon reflection, the elements of the elliptic vibration being often used to calculate the optical constants of the metal. Artificially polished metal surfaces have, however, given very inconsistent results, owing perhaps to films left by the polishing material. Metallic crystals, on the other hand, are usually so small as to render ordinary methods of polariscopic analysis exceedingly difficult to apply.

The present method is a modification of one used originally by Voigt for the identification of elliptically polarized light. The light under examination passes first through an arrangement of quartz wedges acting as a Babinet compensator, then through a "rotator" consisting of another pair of quartz wedges cut perpendicular to the axis, one from right-handed, the other from left-handed quartz; and finally through a large Glan-Nicol prism. The result is that the field is filled with rows of black spots in regular arrangement; and from the location of these spots with reference to the cross-hairs, as photographed, the exact character of the elliptic vibration can be readily calculated. In this particular application the light being studied comes by reflection

[1] Abstract of a paper presented at the Chicago meeting of the American Physical Society, December 1, 1917.

from one facet of a small crystal, and the beam available is therefore very slender, so that only a very small portion of the field is illuminated at once. *In order to produce the spot pattern, the analyzing apparatus is carried back and forth with a sort of weaving motion, at right angles to the beam, until the whole field is covered.* The pattern then appears clearly on the plate, and measurements are easily made upon it. The apparatus used for this purpose is embodied in an instrument for which the name suggested is the *crystelliptometer*.

Excellent plates have been obtained by this method from very small spikelets of selenium and tellurium. The results exhibit beautifully, not only the double refraction of these crystals, but also the continuous transition from one set of optical constants to quite a different set as the crystal is gradually turned with reference to the plane of reflection, the exact nature of which transition has been carefully investigated. The work is being conducted in both the visible and ultra-violet regions.

COE COLLEGE AND UNIVERSITY OF IOWA.

## NEW BOOKS.

*The Nature of Solution.* By HARRY C. JONES. New York: D. Van Nostrand Co., 1917. Pp. xxiii + 380. Price $3.50.

This book, published after the author's death, is not a text-book or treatise, but rather, as indicated in the preface, a semi-popular exposition of the present state of our knowledge of solutions. It is interestingly written, and most of it could be read without difficulty by a student with only an elementary knowledge of physics and chemistry. On the other hand it contains an array of facts, many of which are likely to have been overlooked by physicists, and perhaps chemists also, who have not followed closely the development of physical chemistry.

It seems to the reviewer that it could have been improved by less elaboration in the simpler parts of the subject, and more of it in the more difficult chaptnr on colloidal solutions, which, though long, impresses one as being written rather hastily. There are evidences that on a very few matters the author's physical concepts were not entirely clear, as when (p. 195) he remarks on the great contrast between the numerical values of the speeds of gaseous molecules and the mobilities of ions in solutions, quantities which are too different in character to admit of comparison.

H. M. R.

*Everyday Physics. A Laboratory Manual.* By JOHN C. PACKARD. New York: Ginn and Co., 1917. Pp. vi + 136. Price $1.00.

This is a manual designed for high-school laboratories. The experiments are selected from everyday familiar objects and mechanisms, such as gas and electric stoves, incandescent lamps, heating systems, pressure gauges, water meter, sewing machines, life preservers, etc. On the whole the experiments seem well selected and the plan of the book excellent.

O. M. S.

*The Electron.* By ROBERT ANDREWS MILLIKAN. Chicago: The University of Chicago Press. Pp. xii + 268. Price, $1.50 net.

Occasionally in science as in other fields of human activity a classic appears, that is to say a work which is practically finished and which has a permanent value. It does not seem too much to assign this title to that part of this small volume which describes the author's measurement of the elementary electrical charge, for it seems highly probable that no better method or much higher degree of accuracy of measurement of this fundamental constant will ever be attained. In addition to achieving its principal end, the method has given some important by-products, such as its indication of the limits of validity of Stokes's law; the proof that ionization by X-rays and beta and gamma

rays consists in the detachment of a single electron from a molecule; the verification of Einstein's calculations of the displacement of particles in the brownian movement in gases; and it has given strong evidence in opposition to Thomson's "ether string" theory. In addition to the discussion of the elementary charge there are chapters on early views regarding electricity; the extension of electrolytic laws to conduction in gases; the mechanism of the ionization of gases by X-rays and radium rays; brownian movements in gases; the evidence disproving the existence of a sub-electron; the structure of the atom, and the nature of radiant energy. In all the chapters except the last we have the record of the positive and wonderful achievement of the past twenty years; and in the last chapter there is a clear statement of the great outstanding problem and some suggestions which may be helpful in its solution. The Bohr atom is accepted as giving a correct picture of atomic structure, in spite of its apparent inconsistency with established electromagnetic laws, in the belief that these contradictions will disappear when we reach a clearer conception of the relations between atoms, electrons, and the ether which determine radiation. The text avoids mathematics, but the most important mathematical developments and atomic data are given in appendices. The book is clearly written, and for the most part may be easily followed by any one who has an elementary knowledge of physics. To this class of readers it will give a most interesting and convincing demonstration that atoms and electrons are not the fantasies of visionaries, but realities; and to the professional physicist it offers a well-balanced review and thoughtful criticism of the most important work of recent years.

E. P. L.

*The Mystery of Matter and Energy.* By ALBERT C. CREHORE. New York: D. Van Nostrand Co., 1917. Pp. xi + 161. Price, $1.00.

This little book has for its object the presentation to the general reader of the problem of the structure of matter, with the hope of arousing interest and enthusiasm by making clear the nature and importance of the ends to be attained, and also to give in non-mathematical language the results of the author's speculations and calculations in this field. There are chapters on the atomic constitution of matter; the discovery of the electron and measurement of its charge; electromagnetic waves; the ether and relativity theory; X-rays and atomic numbers. The author describes the results of his calculations, based on the theories of Thomson and of Lorentz, which indicate that gravitation may be explained as the result of forces between revolving electrons, and that the arrangements of atoms in crystals and various crystalline properties may be explained as results of these forces. It is interesting to note that this theory predicts the temperature effect on gravitation which Shaw claims to have discovered. The concluding chapter discusses Planck's quantum theory and suggests an explanation consistent with electromagnetic theory. Much of the subject matter of the book is admittedly hypothetical, but it will be found interesting and suggestive.

E. P. L.